Distributed control & optimization based on internal model

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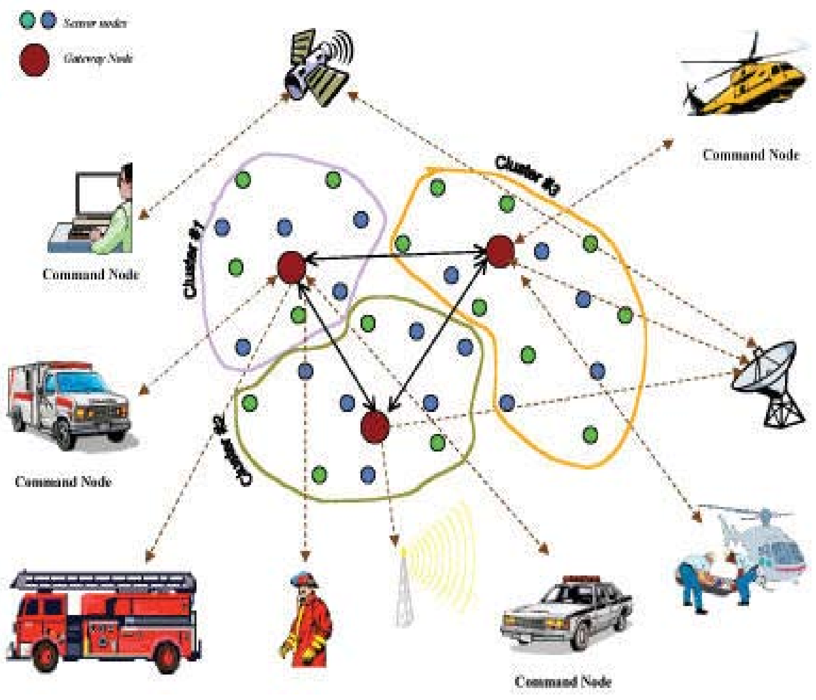
内容

- 背景介绍和基本知识
- 分布式输出调节控制
  - Adaptive internal model
  - Networked internal model
- 分布式抗干扰优化
- 结束语

一、 背景介绍

Multi-agent systems （多智能体、多自主体、多个体系统）

- Dynamics: agent (homogeneous and heterogeneous), environment (passive or active), emergence (split or merge), …;
- Information: directed (agent-agent), indirected (agent-environment), …;
- Control: neighbor-based (switching) rules, mass-based control, partial centralized control, …



多智能体控制和优化问题

- 控制和估计: consensus, containment, formation, flocking, attitude synchronization, Kalman filter, localization , data fusion …
- 优化和覆盖: distributed optimization, sweep, evasion/pursuit, research/rescue, …
- 演化和智能算法: opinion dynamics, social networks, evolutionary or swarm intelligence, evolutionary game, …

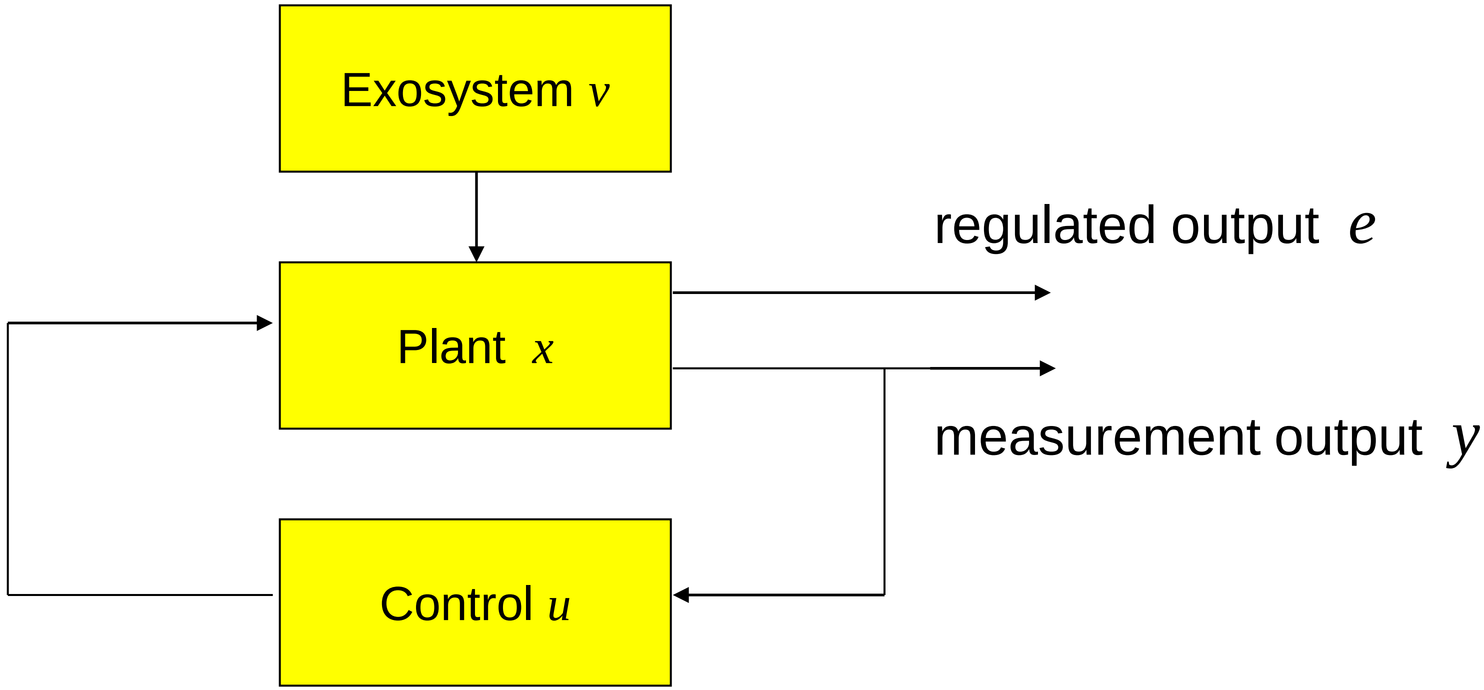
我们的一些成果（控制）

- Containment control & multiple leaders (Automatica 2012; IEEE TAC 2012; Automatica 2014)：一个领导者变成一个集合或者多个领导者
- Distributed output regulation (IEEE TAC 2010, 2013, 2014, IJRNC 2013)：领导跟随的一般框架，包括两个基本方法（内模方法和观测器方法）
- Attitude synchronization (Automatica 2014)：非线性智能体的动力学；给出了一般连通性条件下的两类同步算法及其收敛域
- Target surrounding (IEEE TAC 2014)：一群智能体对一个集合的等距离包围
- Distributed Kalman filter (IEEE TAC 2013)：噪声下的分布式估计

我们的一些成果（优化等）

- Distributed optimization (IEEE TAC 2013; SCL 2013; IEEE TAC 2014)：随机或确定性下的分布式凸优化
- Coverage: cooperative sweeping (Automatica 2013)：对给定区域带有不确定工作量的清扫覆盖问题
- Quantization in control and optimization (CDC 2013, IEEE TCNS conditionally accepted): 量化下的控制与优化，通讯复杂性
- Opinion dynamics (Physica A 2013)：观点（舆论）的演化分组，特别是其波动性的研究
- … …

二、分布式输出调节



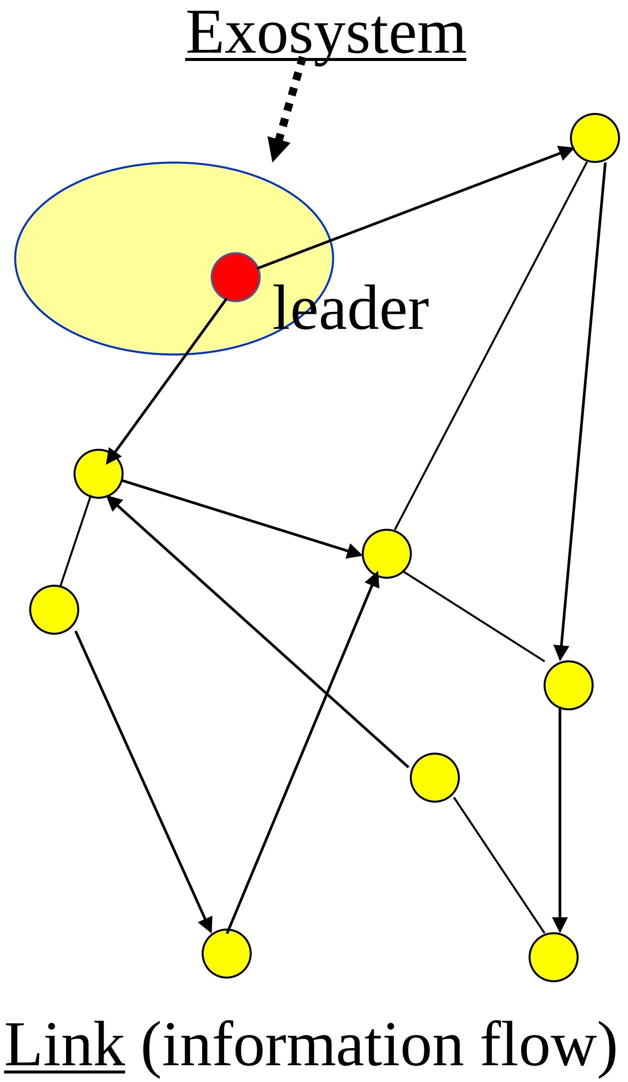
<u>输出调节 Output regulation:</u> $e \rightarrow 0$ *& x is bounded.*

Stabilization, asymptotical tracking, disturbance rejection …

DOR for leader-following consensus

Motivation of distributed output regulation:

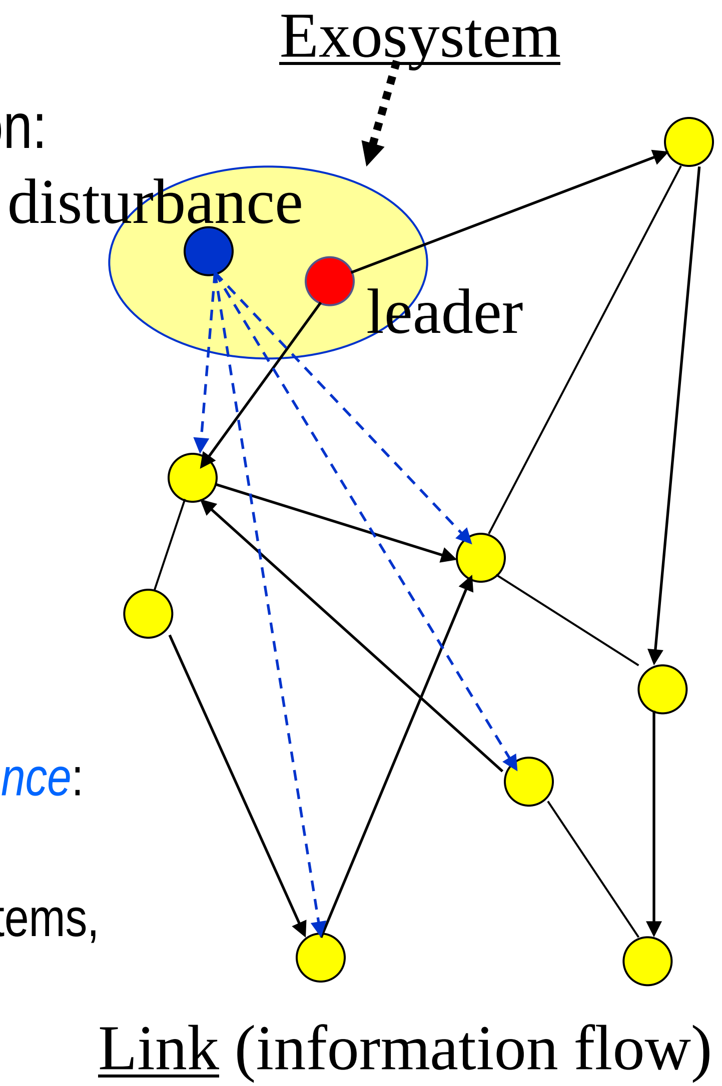
- Provide a general framework for leader-following consensus
  - *Exosystem*: leader (reference signal)
  - *Plant:* followers



DOR beyond leader-following consensus

Motivation of distributed output regulation:

- Provide a general framework for leader-following consensus
  - *Exosystem*: leader
  - *Plant:* followers
- Facilitate the study of more coordination problems
  - *Nonlinearity or Uncertainties or Disturbance*: robust and adaptive control …
  - Multi-level networks: cyber-physical systems, combination of decentralized OR and Distributed OR, host internal model …



Output regulation (OR)

Two main approaches to output regulation:

- Observer/estimator (feedforward): estimate the exosystem and construct regulation feedback

- Internal model (IM): build regulation feedback based on IM → robustness ...

IM for output regulation

- Linear systems (Davison, Wonham, Francis, …): classic IM → incorporate a model of the exosystem (1970's)
- Nonlinear systems (Isidori, Byrnes, Huang, …): Different IMs → incorporate a model determined jointly by both the plant and exosystem (after 1990).

➔ Large-scale systems (Ding, Gazi, …): decentralized OR control → each agent can get the information of the exosystem

Existing Results

Two main approaches to DOR

- **Distributed observer/estimator**: *Hong et al Automatica 2006; Hong, et al Automatica 2008; Hong et al, JSSC 2009 …*
- **Distributed internal model**: *Wang, Hong, et al IEEE TAC 2010; Hong et al Int J Robust & Nonlinear Control, 2013; Su, Hong et al IEEE TAC 2013; Xu, Hong et al, IEEE TAC 2014 …*

DOR: Fundamental challenges

Linear systems:

- Solvability: when DOR can be achieved using local neighbor information
- Connectivity condition: fixed or switched?
- Design: how to give IM-based design for DOR

Nonlinear systems:

- Solvability: necessary/sufficient conditions with given topology?
- Connectivity condition: new communication structure for exchanging local information?
- Design: new IM?

2.1 Adaptive DOR

- More complicated dynamics: nonlinear dynamics and uncertain parameters
- Adaptive DOR control for the exosystem with uncertain parameters → nonlinear control

Nonlinear IM

For nonlinear systems, classic IM does not work → various IMs (canonical IM, etc): incorporate a model determined jointly by both the plant and exosystem

Construct steady-state generator (SSR) for the design of IM

Steady-state generator (SSG)

SSG is a basic step to construct modern IM for nonlinear systems

For exosystem: $dv/dt=S(v)$ and $dx/dt=f(x,u,v,w)$, if there are smooth functions θ, α, β vanishing at $(v,w)=(0,0)$ such that, $\forall\ (v,w)\in V\times W$

$$\frac{d\theta(v,w)}{dt}=\alpha(\theta(v,w)),\ \mathbf{u}(v,w)=\beta(\theta(v,w))$$

where **u** is the solution to the RE .

From SSG to IM

With SSG $\{\theta, \alpha, \beta\}$,

$$\dot{\eta} = \gamma(\eta, e, u)$$

with output u, is an IM candidate if $\forall$ (v, w),

$$\alpha(\theta(v, w)) = \gamma(\theta(v, w), 0, \mathbf{u}(v, w)).$$

The IM candidate becomes an IM if it ensures the stabilizability of the closed loop system.

Some known cases: linear SSG (maybe with nonlinear output map), SSG in uniformly observable form…

Adaptive IM: relatively new

- Adaptive IMs for conventional OR for the exosystem with uncertain parameters: Serrani, Isidori et al, 2001; Marino & Tomei, 2003; Liu, Huang et al, 2009; Obregon, Castillo et al, 2011
- Su & Huang, SCL 2013: relative degree=1, undirected graph, LH adaptive IM
- Our cases: relative degree=r>1, directed graph, OC adaptive IM ($\rightarrow$ containment problem, Automatica, 2014)

Adaptive DOR

The leader contains uncertain parameters → DOR design based on adaptive IM to make $e \rightarrow 0$:

- Leader is linear, $dv/dt=S(\sigma)v$, where σ is the uncertain parameter vector
- Followers may be nonlinear

Design method

Two main design steps: adaptive IM + stabilization of closed-loop systems

- Adaptive IM: find an observable pair based on the solution of RE (polynomial of v)
- Stabilization control: construction of Lyapunov function for the closed loop system

The result is consistent with the case when there is no uncertain parameter.

Results

Adaptive DOR can be solved and the corresponding controller can be constructed for directed graph and minimum phase with relative degree r.

Design: Construct an adaptive IM; Construct stabilization controller based on the construction of a Lyapunov function for directed graph (it is easier if the graph is undirected for the symmetry of the matrix)

Example 1

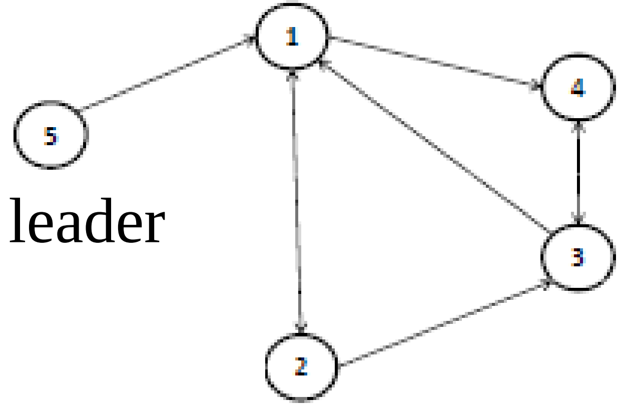
Leader:
ω uncertain

$$\begin{cases} \dot{v}_1 = \omega v_2 \\ \dot{v}_2 = -\omega v_1 \\ y_0 = v_1 \end{cases}$$

4 followers:

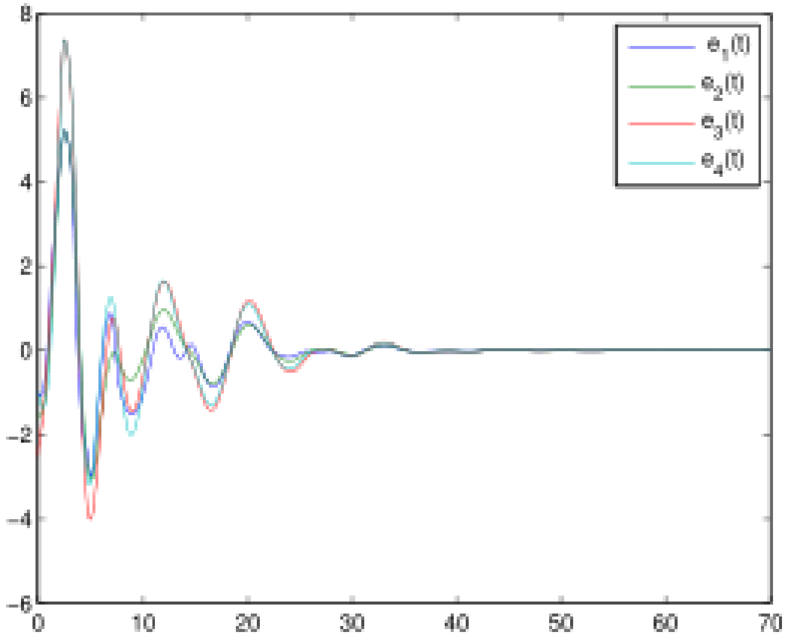
$$\begin{cases} \ddot{y}_i + d_i \dot{y}_i + s_i y_i = u_i \\ 0.5 \le d_i \le 1.5,\ 1 \le s_i \le 2 \end{cases}$$

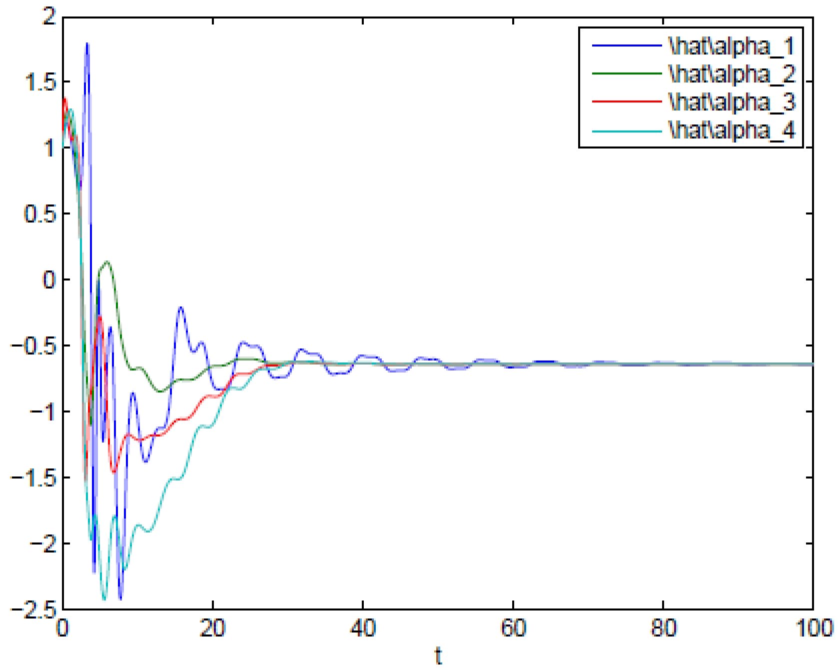
- 4 followers with 1 leader
- Directed graph
- Uncertainties both in leader and followers



leader

Numerical simulations





Tracking errors of followers:
Regulated error y_i-y_0 → 0

Estimation errors of followers:
The estimate → $\alpha_0 = -\omega^2$

2.2 Networked IM

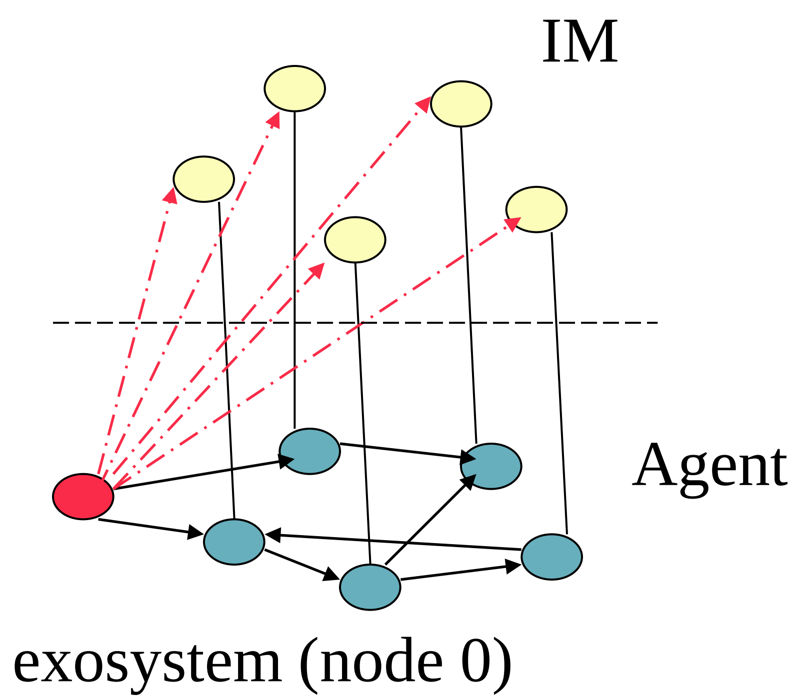
DOR with two-level networks: additional network to exchange IM information (CCC2012, ICARCV 2012, CCC 2013, IEEETAC 2014).

Motivation:

- A framework for distributed OR and decentralized OR (large-scale systems)
- New IM to solve complex network coordination by sharing IM information

One level graph for DOR

- The graph describe the interaction between agents, but sometimes (nonlinear agents …), it is hard to achieve DOR only with the graph
- For DOR, each agent has its own IM for the same exosystem (leader or disturbance source) → what a waste?



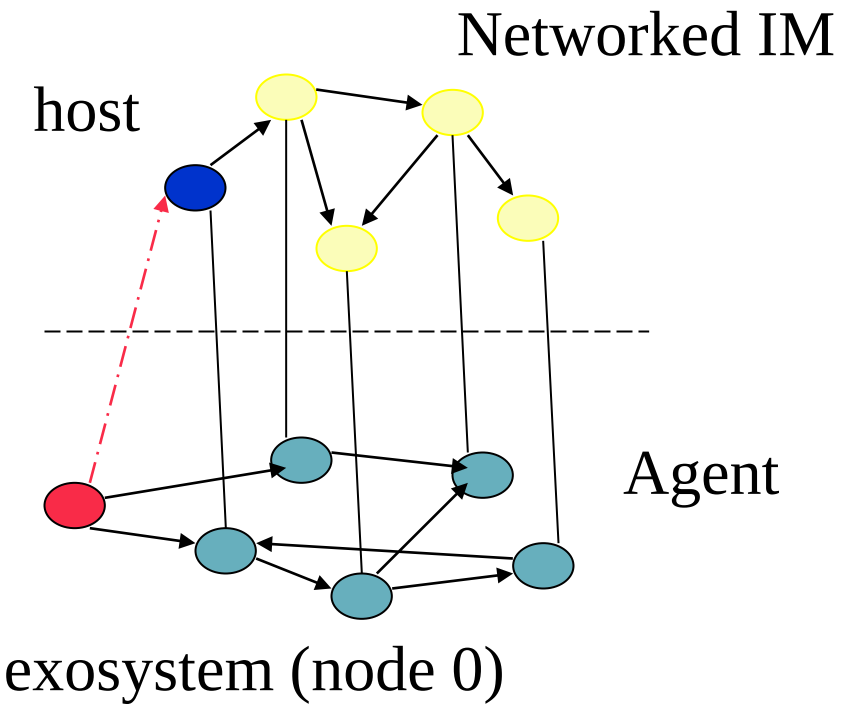
Two-level graph for DOR

Based on 2-level graph:

1. Plant (agent) graph: physical connection between agents & measurement information
2. IM (controller) graph: communication between controllers

Effective design for DOR?

Share IM information, improve performance …



Some agents may not have structural information of node 0

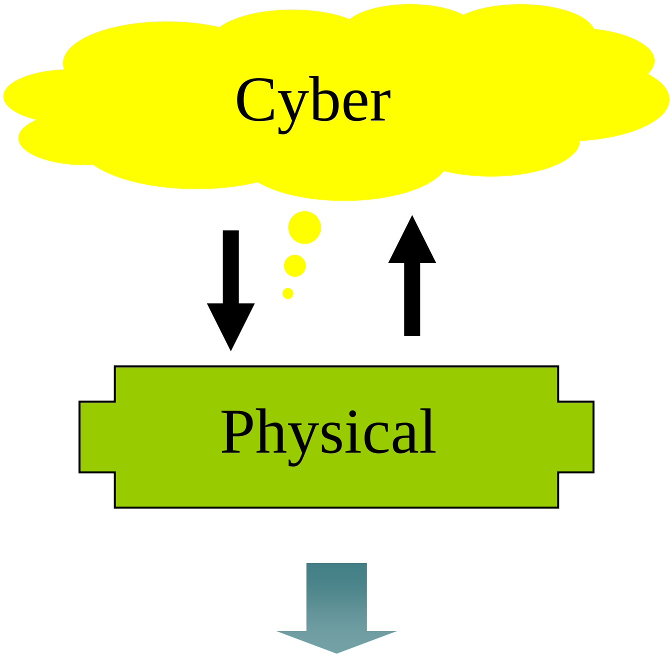
2-level network: cyber-physical ?

Unify decentralized and distributed design for OR:

Large-scale decentralized control on physical layer (fixed plant graph)

+

Distributed control on cyber or communication layer (variable controller graph)



Mathematical analysis for 2-level network …

Networked IM (CCC2012)

DOR Design to share the information of neighbor IM-based controllers.

IM → networked IM:

$$\dot{\eta}_i = f_i(\eta_i, x_i, \eta^\natural) \quad \text{or} \quad \eta_i = g_i(\eta^\natural)$$

where

$$\eta_i^\natural = (\eta_j, j \in \mathcal{O}_i^c)$$

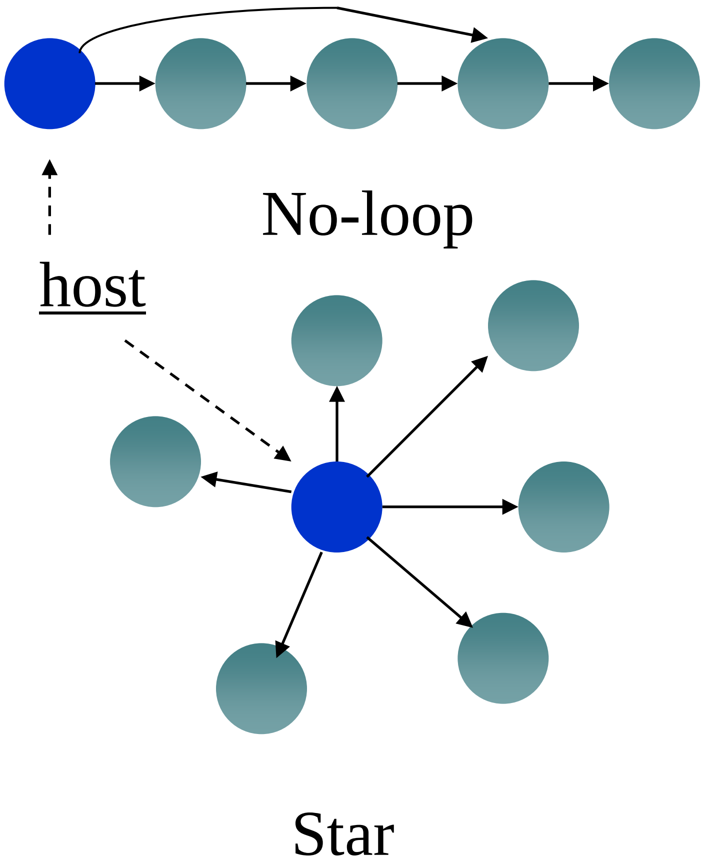
Neighbor information

The distributed controller: $u_i = h_i(e_i, \eta_i)$

Measurement variable IM variable

Challenges in IM network design

- Construction of IM
- Structure of IM network (different from the agent network): 2 simple cases
- Selection of host IM for homogenous or heterogeneous agents
- Solution of the RE and construction of Lyapunov function
- Nonlinear and uncertain agents/leaders …



Example 2

The exosystem consists of two parts: leader & disturbance

- Leader: Lienard system

  y_0 : output of the leader

$$\begin{cases} \dot{v}_{r1} = -G(v_{r2}), \\ \dot{v}_{r2} = v_{r1} - F(v_{r2}) \\ y_0 = v_{r1} \end{cases}$$

- Disturbance model:

$$\begin{cases} \dot{v}_{d1} = v_{d2}, \\ \dot{v}_{d2} = -\omega^2 v_{d1} \\ \dot{v}_{d3} = 0 \end{cases}$$

Heterogeneous followers

Agent 1 with x_1

$$\dot{x}_1 = u_1,$$

$$e_1 = x_1 - y_0$$

Agent 2 with x_2

$$\dot{x}_2 = u_2 + x_1^2 - v_{d1},$$

$$e_2 = x_2 - x_1.$$

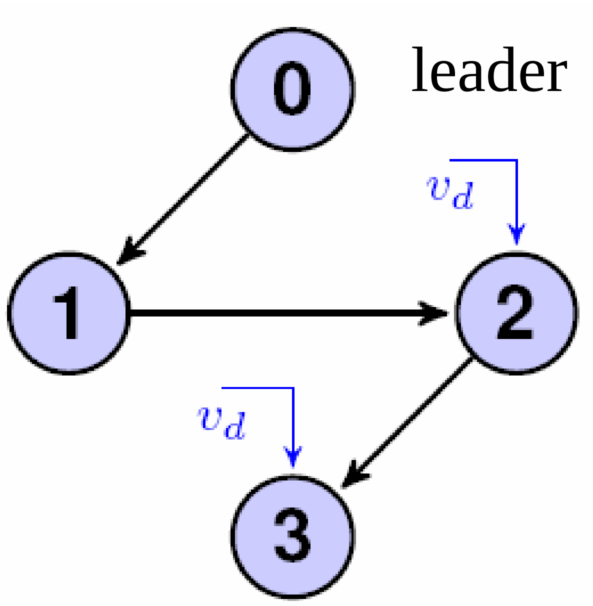
Agent 3 with x_3=col(z_3 y_3)

$$\dot{z}_3 = -z_3 + e_3 y_3,$$

$$\dot{y}_3 = u_3 - v_{d3} v_{d1} + z_3^2 + v_{d2}^3 - \sin^2(x_2)$$

$$e_3 = y_3 - x_2.$$

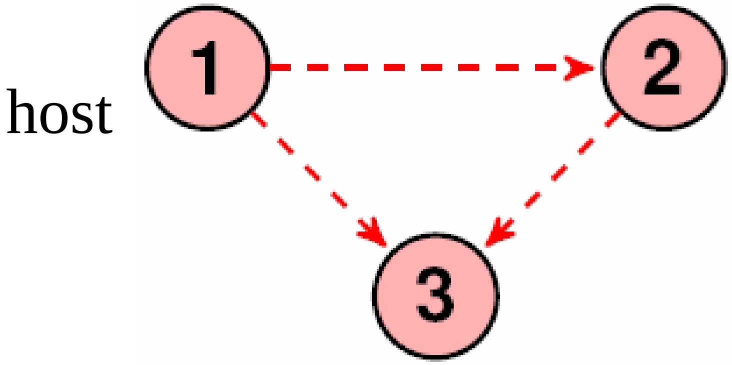
Plant/agent graph:



Relative measurement e_i

IM/Controller graph

IM/controller graph:



Cooperative controller based on networked IM: (cyber) IM variable η_i exchanged over the controller graph

- Helpful to solve RE
- Largely reduce structural complexity of IMs for agents 2 and 3,…

Results (ICARCV 2012)

With more information exchanges, solve the unsolvable problems in decentralized control, and reduce the complexity of IM-based controller.

There are solution of RE and SSR for heterogeneous nonlinear agents in output-feedback form → networked IM can be constructed to be input-to-state stable to solve DOR for fixed two-level graphs without loops.

Results (ICCA2014, IEEE TAC2014...)

Leader

$$\dot{v}_r = S_r v_r, \;\; y_0 = q(v_r, w)$$

Follower

$$i \in \mathcal{O}: \quad \begin{cases} \dot{z}_i = f(z_i, y_i, w) \\ \dot{y}_i = g(z_i, y_i, w) + \delta_i(v_i) + u_i \end{cases}$$

- $\mathcal{O} := \{1, \cdots, N\}$
- strict-feedback uncertain system having unity relative degree

Disturbance

$$i \in \mathcal{O}: \quad \dot{v}_i = S_i v_i$$

Problem Formulation

Two outputs

- Regulated output: control aim, unavailable for control design

$$i \in \mathcal{O}: \quad e_i = y_i - y_0$$

- Measurement output: available for control design

$$i \in \mathcal{O}: \quad e_{mi} = \sum_{j=0}^{N} a_{ij}(y_i - y_j)$$

Semi-global leader following

For any index $\jmath$ and any sets $\mathcal{B}_\rho^z$ and $\mathcal{B}_\rho^y$ with

$$z := (z_1, \cdots, z_N), \; y := (y_1, \cdots, y_N)$$

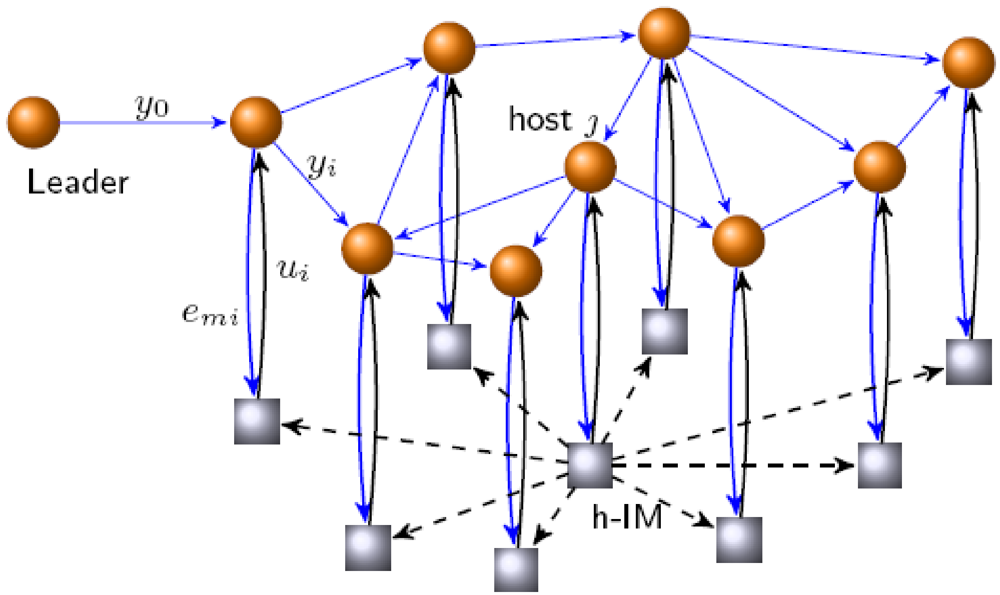
find a smooth distributed controller

$$\begin{cases} \dot{\eta}_\jmath = h_\eta(\eta_\jmath, u_\jmath), \; \dot{\xi}_i = h_{\xi i}(\xi_i, \eta_\jmath, u_i) \\ u_i = u_{ci}(\xi_i, \eta_\jmath, e_{mi}), \; i = 1, \cdots, N \end{cases}$$

with a set $\mathcal{B}_{\rho'}^{\xi'}$, where $\xi' = (\eta_\jmath, \xi_1, \cdots, \xi_N)$, such that, for any initial conditions in set $\mathcal{B}' := \mathbb{V} \times \mathbb{W} \times \mathcal{B}_\rho^z \times \mathcal{B}_\rho^y \times \mathcal{B}_{\rho'}^{\xi'}$,

1. the trajectory of the closed-loop exists and is bounded;
2. $\lim_{t\to\infty} e(t) = 0$

Host IM



Host agent: 2 IMs:

- One IM (host IM) is to track the leader
- local one is to reject local disturbance

Other agents : local IM to reject local disturbance

Main Result

Theorem: Under standard assumptions, for any slected host agent index $\jmath$, the semi-global leader-following consensus problem can be solved by the host-internal-model-based control

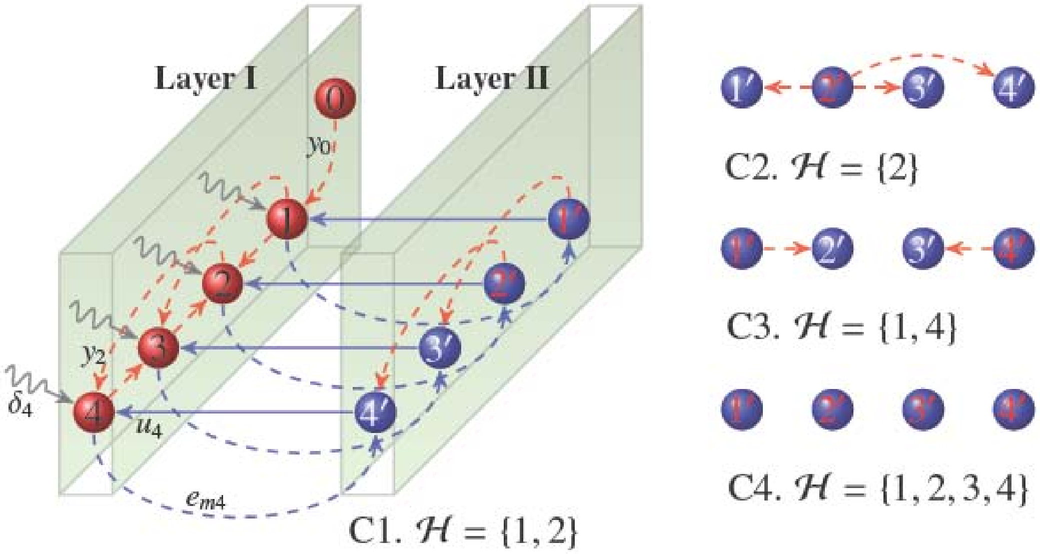
$$i \in \mathcal{O}: \quad \dot{\eta}_i = \begin{cases} M_i \eta_i + \Gamma_i u_i, & i = \jmath \\ M_i \eta_i + \Gamma_i (u_i - \bar{\Psi}_r \eta_\jmath), & i \neq \jmath \end{cases}$$

and stabilization control

$$i \in \mathcal{O}: \quad \bar{u}_i = -k e_{mi}$$

Remarks

- A striking reduction of the controller order and computing burden, while a one-dimensional signal is transmitted
- More general IM network can also be constructed. For example



Example 3

Consensus design of a group of FitzHugh-Nagumo type agents with local disturbances

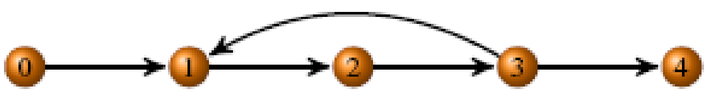
Leader $$\dot{v}_{r1} = \omega_r v_{r2},\ \dot{v}_{r2} = -\omega_r v_{r1},\ y_r = v_{r1}$$

Follower
$$\begin{cases} \dot{z}_{i1} = \sigma_5(y_i - \sigma_2 z_{i1}) \\ \dot{z}_{i2} = \sigma_6(-y_i - \sigma_4 z_{i2}) \\ \dot{y}_i = y_i - \frac{1}{3}y_i^3 - z_{i1} + z_{i2} + F_i(t) + u_i,\ i = 1, \cdots, 4 \end{cases}$$

Disturbance $$F_i(t) = A_{mi}\sin(\omega_i t + \phi_i) + d_i$$

Simulation

Measurement graph



Host agent index

$$j = 2$$

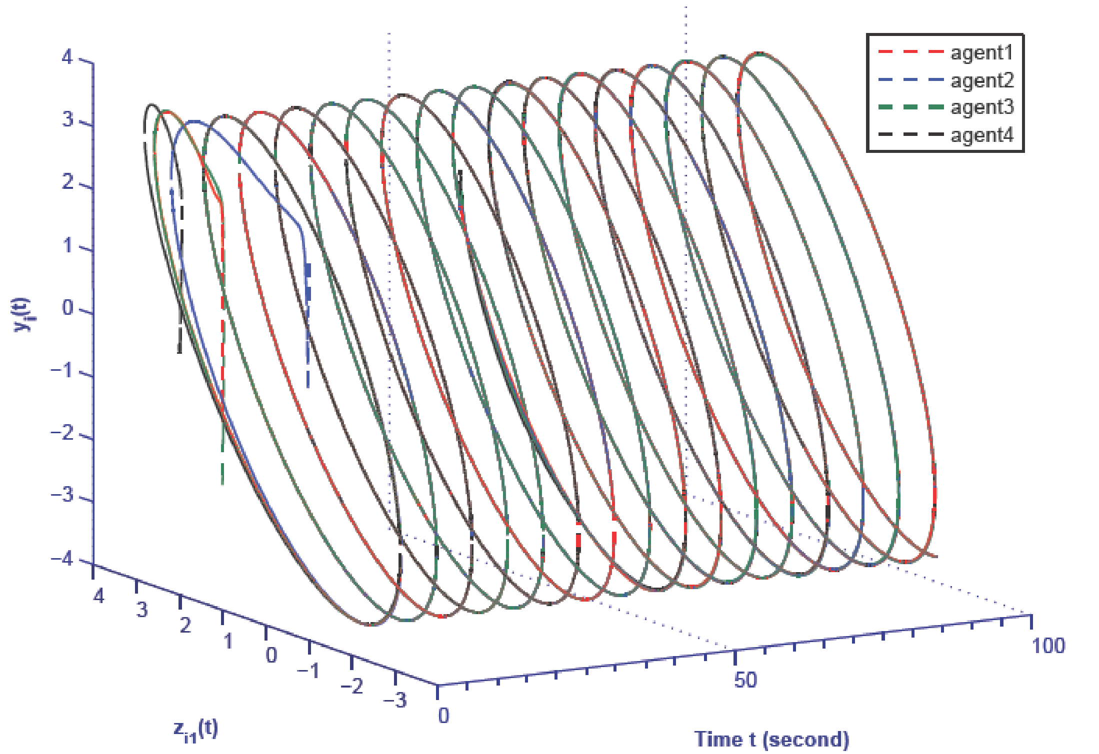


Figure 2: 3D plot of agent responses with axis (t, z_{i1}, y_i).

三、 分布式抗干扰优化

Convex optimization: $\min_{z \in R^m} \quad f(z)$

→ Distributed convex optimization:

$$\min_{z \in R^m} \quad f(z) = \sum_{i=1}^{n} f_i(z)$$

Extensions:

- Zero-sum game: minmax $f(x,y)$
- Constrained convex optimization → Non-convex optimization

Basic motivation and idea

- 现有的分布式优化设计没有考虑不确定性对优化过程的影响
- 当要实现分布式优化的个体为现实为物理实体（如机器人、无人车等）时，各种干扰不得不要考虑
- 在干扰下实现精确分布式优化：连续时间个体动力学 + 分布式凸优化 + 由一个外在系统产生的干扰

基本假设

- 优化函数严格凸和局部Lipschitz
- 固定连通图
- 干扰的频率已知

两种两种情况：

- 无向图
- 有向平衡图 + m-Lipschitz

Problem setup

Consider the following system

$$\dot{x}_i = u_i + d_i(t), \quad i = 1, \cdots, N$$

where x_i is the state, u_i is the input, and d_i is the disturbance governed by

$$\dot{w}_i = Sw_i, \quad d_i = Cw_i(t)$$

The control aim is to find distributed control

$$\dot{z}_i = g_{i1}(z_i, \nabla f_i(x_i), x_{mi})$$
$$u_i = g_{i2}(z_i, \nabla f_i(x_i), x_{mi})$$

to solve the optimization problem: $x_i \rightarrow x^*$ with

$$x^* = \arg\min_{x \in \mathbb{R}^n} f(x), \quad -\infty < x^* < +\infty$$

抗干扰分布式优化算法

控制器：优化的次梯度项 + 个体协作的趋同项 + 基于内模的抗干扰项。

$$\dot{v}_i = \alpha\beta \sum_{j=1}^{N} a_{ij}(x_i - x_j)$$

$$\dot{\eta}_i = (I_n \otimes F)\eta_i + (I_n \otimes G)u_i$$

$$u_i = \underbrace{-\alpha\nabla f_i(x_i) - v_i}_{\text{optimal term}} \underbrace{-\beta \sum_{j=1}^{N} a_{ij}(x_i - x_j)}_{\text{consensus term}}$$

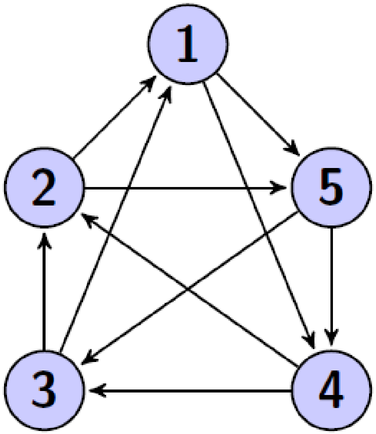
$$\underbrace{-(I_n \otimes \Psi)\eta_i}_{\text{internal model term}} .$$

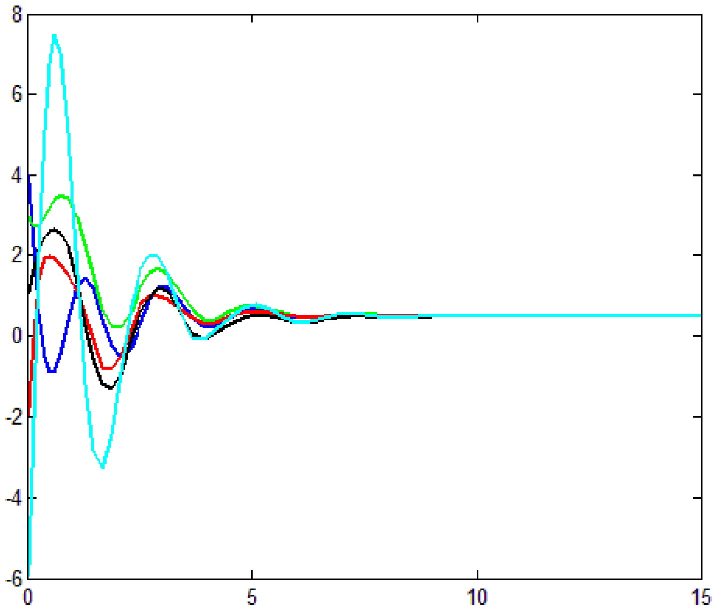
主要结果

- 在已有假设下，抗干扰精确优化可以全局实现（Wang, Yi, Hong, Control Theory & Technology, 2014）
- 如果干扰频率未知，可采用自适应内模使得抗干扰精确优化半全局实现（Wang, Hong, Yi, CCC 2014; 注意到优化函数和未知频率都使得系统变成非线性）
- 结果可以推广到二阶甚至高阶（线性）个体动力学情况(ongoing work)

Example 4（5 agents）

连通拓扑图和优化误差曲线





$$f_1(x) = (x+2)^2, \quad f_2(x) = (x-5)^2$$

$$f_3(x) = x^2 \ln(1+x^2) + x^2$$

$$f_4(x) = \frac{x^2}{\sqrt{x^2+1}} + x^2, \quad f_5(x) = \frac{x^2}{\ln(2+x^2)}$$

4 Conclusions

MAS: collective dynamics and distributed algorithms

- When conventional theory (e.g., output regulation or optimization) meets new topic (e.g., MAS) → Distributed output regulation or optimization: framework, fundamental problems, …
- MAS: many new topics beyond consensus

Thank you!