Cooperative Control of Multi-agent Systems: A Distributed Observer Approach

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2014 Workshop on Distributed Coordinated Control of Dynamic Multi-Agent Systems
July 23-25, 2014

Acknowledgement

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Outline

1. Introduction

Collective Behaviors

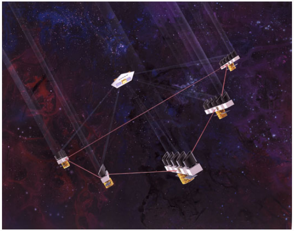
School of Fish
(S. Martinez, et al. 2007)

Flocking of Birds
(http://www.fws.gov)

Swarm of Locusts
(http://sciencephoto.com/image)

Multi-agent Systems

Robot Formation
(http://www-symbiotic.cs.ou.edu)



Formation of Spacecraft
(http://www.acsu.buffalo.edu)

Flight Formation
(http://4.bp.blogspot.com)

Distributed Control of Multi-Agent Systems (Cont.)

- The individual subsystems can only access the information of their neighbors. Thus the system has to be controlled by a distributed control protocol featuring the so-called nearest neighbor rule.
- Information has to be shared among individual agents, and all agents in the group have a common objective leading to collective behaviors.
- The global behavior of the system is jointly dictated by the system dynamics and the communication topology.
- A basic control problem for multi-agent systems is (leader-following) consensus.

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Consensus

- The leader-following consensus problem is to design a distributed feedback control law such that the outputs of all agents converge to a prescribed trajectory which is usually produced by another agent called leader.
- So far, the consensus problem has been mainly studied for linear, homogeneous multi-agent systems without subjecting to model uncertainty and external disturbances.
- Other variants of the consensus problem include synchronization, flocking, swarming, formation, rendezvous (Fax and Murray, 2004), (Jadbabaie, Lin, Morse, 2003), (Ren and Beard, 2008), etc.

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Cooperative Output Regulation

- The output regulation problem aims to deal with the asymptotic tracking and disturbance rejection problem in an uncertain plant. This objective includes the consensus as a special case.
- The cooperative output regulation problem handles the asymptotic tracking and disturbance rejection problem for uncertain multi-agent systems via a distributed control scheme. Two approaches, namely, distributed internal model based approach and distributed observer approach have been developed since 2009.
- An application of the main result will lead to the solution of the leader-following consensus problem for a nonlinear heterogeneous multi-agent system subject to model uncertainty and external disturbances.
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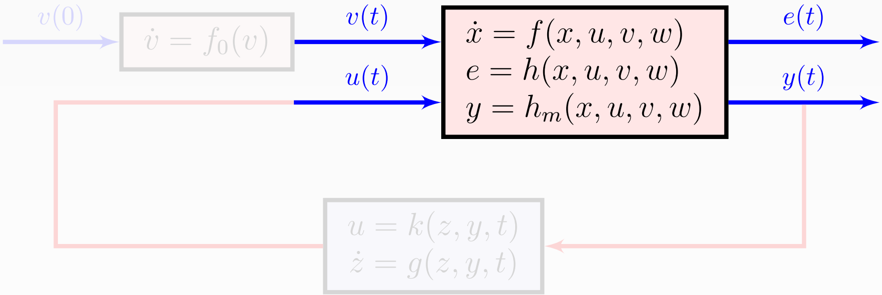
2. Classical Output Regulation:

Feedforward Control Approach

Problem Description



- Problem Statement: Design a control law such that, for all $v(t)$, the solution of the closed-loop system is globally bounded, and satisfies

$$\lim_{t\to\infty} e(t) = 0.$$

- The control law is called measured output feedback control. It includes error output feedback with $y = e$ and full information feedback with $y = (x, v)$ as two special cases.

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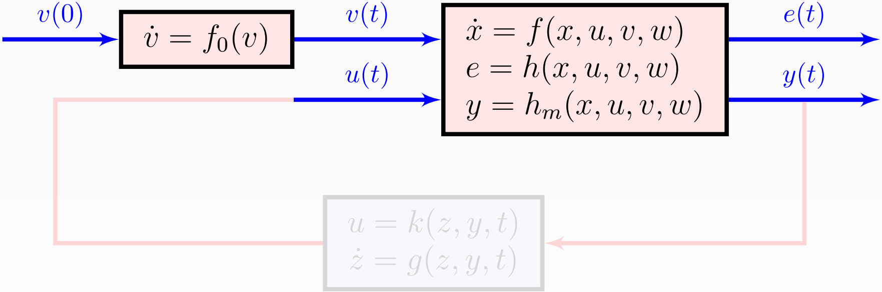

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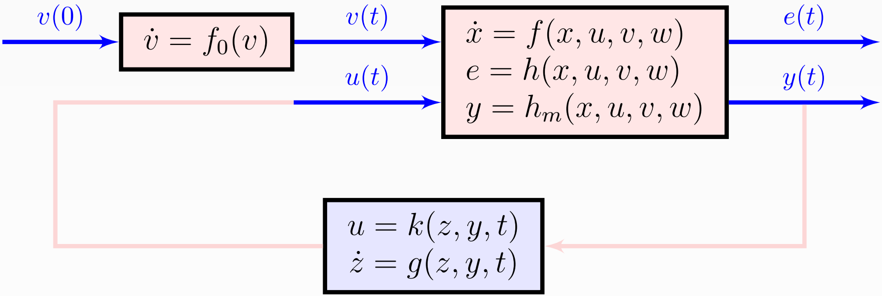

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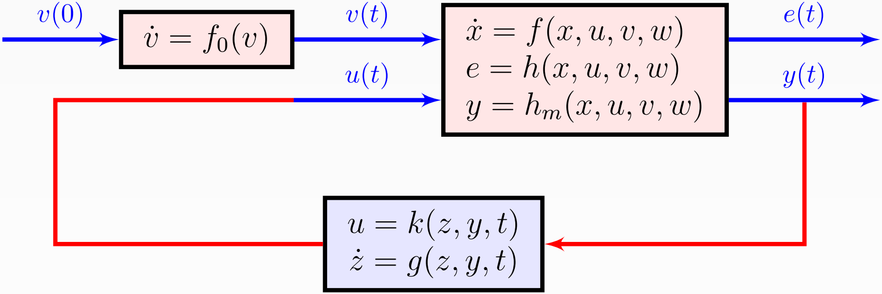

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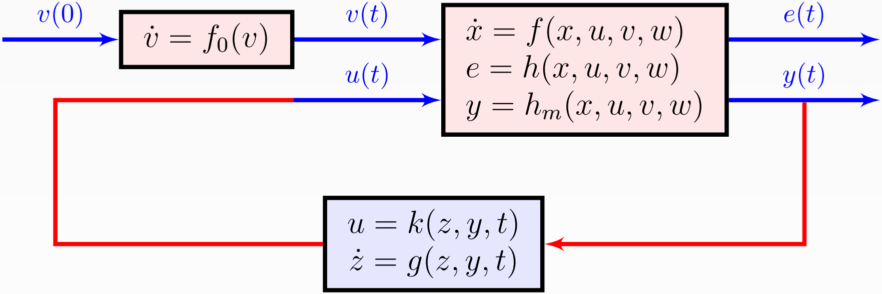

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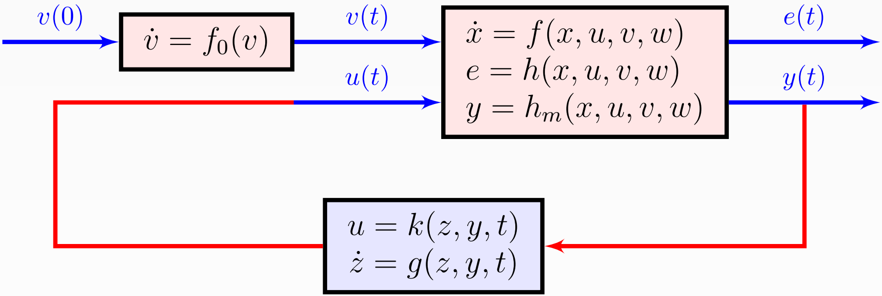

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- The problem handles asymptotic tracking and disturbance rejection simultaneously where both the reference input and disturbances are generated by the autonomous system $\dot{v} = f_0(v)$ called exosystem.
- The problem can be viewed as a leader-following consensus problem with the exosystem as the leader and the controlled plant as the single follower.
- Two different approaches, namely, feedforward control and internal model control have been developed (Isidori, Huang, Khalil, et al).
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Feedforward Control Approach

- The feedforward control assumes $y = (x, v)$. The control law is of the following form:

$$u = k(x, v) \tag{1}$$

which leads to the following closed-loop system:

- The solvability of the output regulation problem by feedforward control approach has been well understood after two decades of research (Isidori, Byrnes, Huang, et al).

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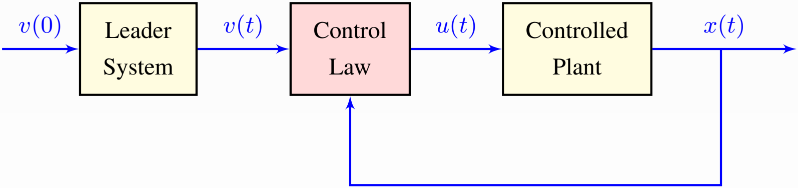

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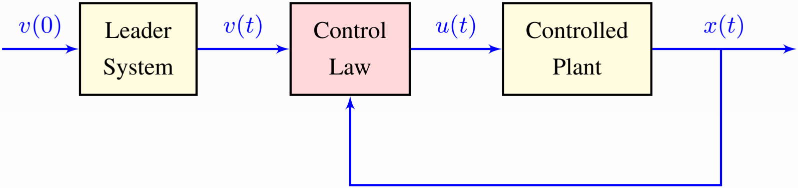

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Observer based Control

- The state $v(t)$ of the exosystem is often not available for control. It is desirable to design a control law which only depends on the measured output of the exosystem.
- Given a system of the form

$$\dot{v} = f_0(v), \quad y_o = g_o(v) \tag{2}$$

where y_o is the measured output of (2). The following system

$$\dot{\eta} = \phi(\eta, y_o) \tag{3}$$

is called an asymptotic observer of (2) if, for any $v(0)$ and $\eta(0)$,

$$\lim_{t\to\infty} (\eta(t) - v(t)) = 0 \tag{4}$$

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Certainty Equivalence Principle

- If an observer based control law (5) solves the same problem as the feedforward control law (1) does, then this control law is said to satisfy certainty equivalence principle.
- For linear time-invariant systems, an asymptotic observer exists generically, and the certainty equivalence principle always holds.
- For time-varying systems or nonlinear systems, an asymptotic observer may not exist, and even if it exists, the certainty equivalence principle may not hold. This is one of the reasons that makes the control of nonlinear or time-varying systems challenging.

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3. Cooperative Control of Multi-Agent Systems

Multi-agent Systems

$$\begin{aligned} \dot{x}_i &= f_i(x_i, u_i, v, w) \\ e_i &= h_i(x_i, u_i, v, w) \quad , \quad i = 1, ..., N \\ y_i &= h_{mi}(x_i, u_i, v, w) \end{aligned} \tag{6}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $e_i \in \mathbb{R}^p$, and $y_i \in R^{p_i}$. The exogenous signal $v \in \mathbb{R}^q$ is generated by the following exosystem:

$$\dot{v} = f_0(v),\ y_0 = h_0(v) \tag{7}$$

- System (6) together with (7) can be viewed as a multi-agent system of $N+1$ agents where the exosystem (7) is viewed as the leader, and all subsystems of system (6) are viewed as N followers.
- If all followers can access the state v of the leader, then the output regulation problem of (6) and (7) can be handled by a so-called decentralized control scheme.

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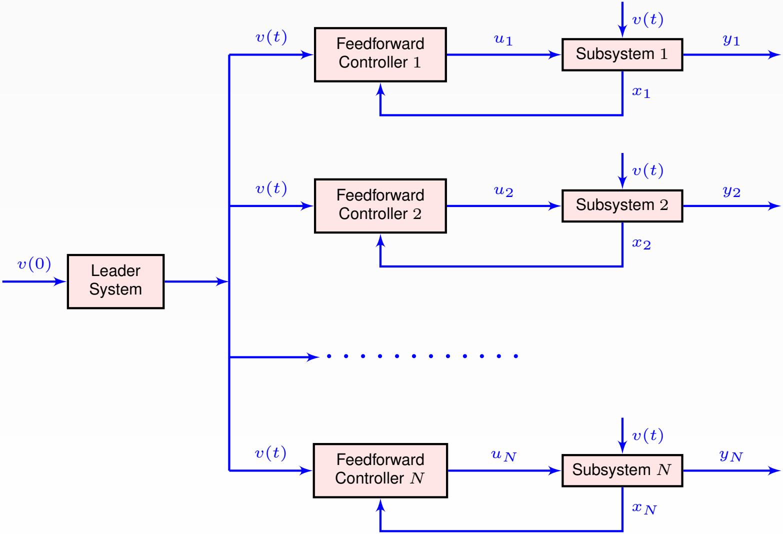
Decentralized Feedforward Control



Communication Graph

- Given the multi-agent system (6) and (7), one can define a communication graph $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$ with $\mathcal{V}$ being the node set and $\mathcal{E}(t)$ being the edge set.
- $\mathcal{V} = \{0, 1, \ldots, N\}$ with the node 0 associated with (7) and the other N nodes associated with the N followers of (6).
- For any $t \geq 0$, $\mathcal{E}(t) \subset \mathcal{V} \times \mathcal{V}$. $(j, i) \in \mathcal{E}(t)$, $i \neq j$, $i, j = 0, 1, \ldots, N$, if and only if the control u_i of the subsystem i, $i = 1, \ldots, N$, can access y_j at time t, $j = 0, 1, \ldots, N$. j is said to be a neighbor of i at time t.
- $\mathcal{N}_i(t) = \{j, \ (j, i) \in \mathcal{E}(t)\}$ denotes the neighbor set of the node i at time t.
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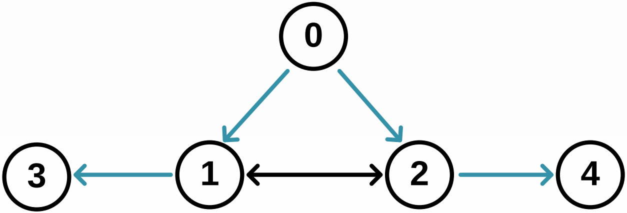
Graph (Cont.)

- A subset of $\mathcal{E}(t)$ of the form $\{(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)\}$ is called a path of $\mathcal{G}(t)$ from i_1 to i_k at time t, and it is said that the node i_1 can reach the node i_k at time t.

- If, at some t, the node 0 can reach all other nodes, then the graph $\mathcal{G}(t)$ is said to be connected at time t.



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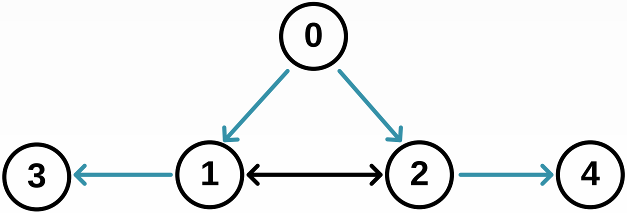

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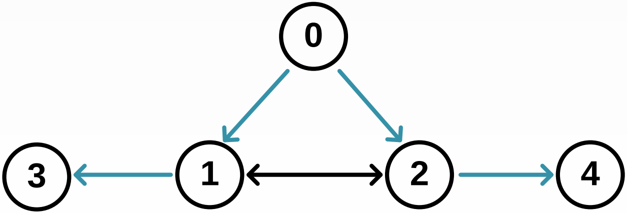

Distributed Control Laws

- Distributed control law:

$$
\begin{aligned}
u_i &= k_i(z_i, y_i, y_j, j \in \mathcal{N}_i(t)) \\
\dot{z}_i &= g_i(z_i, y_i, y_j, j \in \mathcal{N}_i(t)), \quad i = 1, ..., N
\end{aligned} \tag{8}
$$

where $y_0 = h_0(v)$, k_i and g_i are some sufficiently smooth functions.

- Control law (8) satisfies the communication constraints: the i^{th} control u_i depends on y_j iff the agent j is a neighbor of the agent i.

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Problem Formulation

➢ **Definition:** Given the plant, the exosystem, and the graph $\mathcal{G}(t)$, find a distributed control law such that, for any initial condition, the solution of the closed-loop system is bounded, and the error output satisfies

$$\lim_{t\to\infty} e_i(t) = 0, \quad i = 1, ..., N.$$

➢ **Remark 1:** The degree of the difficulty of the problem not only depends on the dynamics of the system, but also the property of the graph $\mathcal{G}(t)$ which can be static or time-varying satisfying such conditions as every time connected, frequently connected, or jointly connected.

➢ **Remark 2:** The jointly connected condition is the most challenging one since such a graph is discontinuous and can be disconnected at any time instant.

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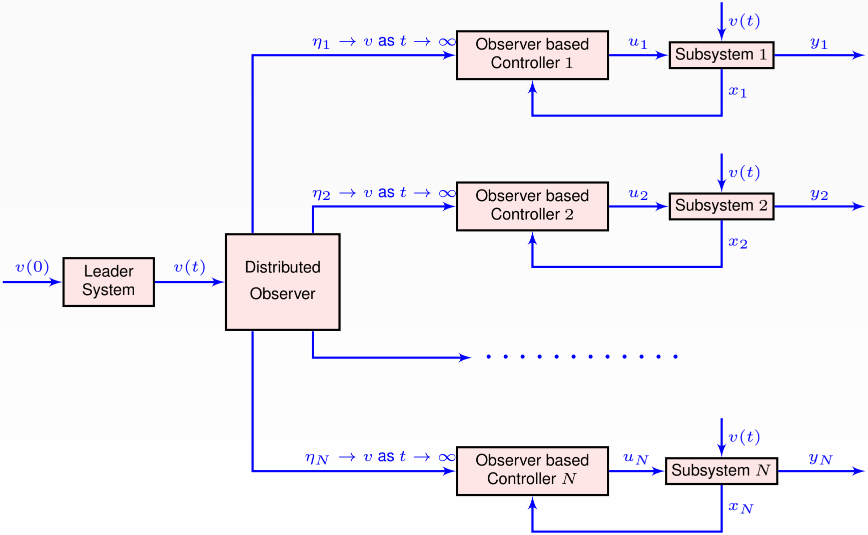
4. A Distributed Observer Approach

A Distributed Observer Based Scheme



Two Technical Issues

- Does such a distributed observer exist?
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Distributed Observer Candidate

- Given the leader system $\dot{v} = f_0(v),\ y_0 = h_0(v)$ and a graph $\mathcal{G}(t)$ with $N+1$ nodes, for $i = 1, \ldots, N$, $j = 0, 1, \ldots, N$, let $a_{ij}(t) > 0$ if $j \in \mathcal{N}_i(t)$, and $a_{ij}(t) = 0$ if otherwise. Then the following compensator

$$\dot{\eta}_i = f_0(\eta_i) + \mu \left(\sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(\eta_j - \eta_i) \right),\ i = 1, \ldots, N \quad (9)$$

where $\mu > 0$, $\eta_0 = y_0$, is called a distributed observer candidate of the leader system, and is called a distributed observer of the leader if

$$\lim_{t \to \infty} (\eta_i(t) - v(t)) = 0, \quad i = 1, \ldots, N \quad (10)$$

- Whether or not (9) is a distributed observer depends on both the dynamics of the leader and the property of the graph.

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Results on Linear Multi-agent Systems

➢ **Proposition 1:** Suppose the leader system is linear, and marginally stable, and the graph $\mathcal{G}(t)$ is jointly connected. Then there exists positive μ such that, for all $v(0), \eta_i(0), i = 1, \cdots, N$, the solution of (9) satisfies

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Thus, (9) is a distributed observer of the leader.

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➢ **Multiple Euler-Lagrange Systems (Li and Slotine, 1991):**

$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, \ i = 1, \ldots, N \tag{13}$$

where $q_i, \dot{q}_i \in R^n$ are the generalized position and velocity vectors, and $\tau_i \in R^n$ is the control input.

➢ **Leader System:**

$$\dot{v} = Sv, \ \ q_0 = Fv \tag{14}$$

where $v \in R^m$, $q_0 \in R^n$, $S \in R^{m\times m}$ and $F \in R^{n\times m}$.

➢ **Remark 4:** System (14) can generate a large class of leader signals such as step function of arbitrary magnitude, ramp function of arbitrary slope, and sinusoidal function of arbitrary amplitude and initial phase.

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$$\dot{\hat{q}}_i = \frac{1}{2}\hat{q}_i^{\times}\omega_i + \frac{1}{2}\bar{q}_i\omega_i, \ \ \dot{\bar{q}}_i = -\frac{1}{2}\hat{q}_i^{T}\omega_i \tag{15a}$$

$$J_i\dot{\omega}_i = -\omega_i^{\times}J_i\omega_i + u_i, \ i = 1, \ldots, N \tag{15b}$$

where $\hat{q}_i \in R^3$ and $\bar{q}_i \in R$, and $q_i = [\hat{q}_i, \bar{q}_i]^T$ is the unit quaternion representing the attitude of the i^{th} rigid body, $\omega_i \in R^3$ is the angular velocity of the i^{th} rigid body, and $J_i \in R^{3\times 3}$ and $u_i \in R^3$ denote the inertia matrix and the control torque of the i^{th} rigid body, respectively.

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➢ Let $v = (q_0, \omega_0)$, and

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Then the leader system takes the following standard form

$$\dot{v} = f_0(v), \ y_0 = v \tag{17}$$

➢**Proposition 2:** Given a nonlinear leader system of the form (17) and a static graph $\mathcal{G}$, there exists positive μ such that, for all $v(0), \eta_i(0)$, $i = 1, \cdots, N$,

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➢ **Theorem 3:** Given a static graph $\mathcal{G}$, the certainty equivalence principle holds for the multiple rigid body system iff the graph $\mathcal{G}$ is connected.

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5. A Case Study: Attitude Consensus of Multiple Spacecraft Systems

Certainty Equivalence Controller

- True Tracking Error:

$$\epsilon_i = q_0^{-1} \odot q_i$$
$$\hat{\omega}_i = \omega_i - C_i \omega_0$$

The error signals ϵ_i and $\hat{\omega}_i$ are not available for every follower.

- Estimated Tracking Error: Partition $\eta_i = [\zeta_i^T, \xi^T]$ with $\alpha_i \in R^4$. Then

$$e_i = \zeta_i^{-1} \odot q_i \quad (18a)$$
$$\bar{\omega}_i = \omega_i - \hat{C}_i \xi_i \quad (18b)$$

where $\hat{C}_i = (\bar{e}_i^2 - \hat{e}_i^T \hat{e}_i) I_3 + 2\hat{e}_i \hat{e}_i^T - 2\bar{e}_i \hat{e}_i^\times$

- Theorem 3 implies that, $\forall\ i = 1, \ldots, N$,

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Certainty Equivalence Controller

- Decentralized Controller:

$$u_i = \omega_i^{\times} J_i \omega_i - J_i(\hat{\omega}_i^{\times} C_i \omega_i - C_i S \omega_i) - k_{1i}\hat{\epsilon}_i - k_{2i}\hat{\omega}_i, \quad i = 1, 2, \cdots, N$$

where $k_{1i}, k_{2i} > 0$.

- Certainty Equivalence Controller:

$$u_i = \omega_i^{\times} J_i \omega_i - J_i(\bar{\omega}_i^{\times} \hat{C}_i \xi_i - \hat{C}_i S \xi_i) - k_{1i}\hat{e}_i - k_{2i}\bar{\omega}_i \quad (19)$$

$$\dot{\eta}_i = f_0(\eta_i) + \mu \left(\sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(\eta_j - \eta_i) \right), \; i = 1, \ldots, N \quad (20)$$

- J_i is uncertain due to
  - uncertain mass distribution;
  - fuel consumption;
  - spacecraft reconfiguration;

  Therefore, the control law u_i must not rely on J_i. Adaptive control is such a control scheme.

The Error System

- To simplify the closed-loop system analysis, performing on (18a) the following transformation

$$\tilde{\omega}_i = \bar{\omega}_i + k_{i1}\hat{e}_i$$

where $k_{i1} > 0$, leads to the following error system:

$$\dot{\hat{e}}_i = \frac{1}{2}(\hat{e}_i^{\times} + \bar{e}_i I_3)(\tilde{\omega}_i - k_{i1}\hat{e}_i) + \alpha_i(t) \quad \text{(21a)}$$

$$\dot{\bar{e}}_i = -\frac{1}{2}\hat{e}_i^T(\tilde{\omega}_i - k_{i1}\hat{e}_i) + \beta_i(t) \quad \text{(21b)}$$

$$\begin{aligned} J_i\dot{\tilde{\omega}}_i = & -\omega_i^{\times} J_i\omega_i + J_i((\tilde{\omega}_i - k_{i1}\hat{e}_i)^{\times}\hat{C}_i\xi_i - \hat{C}_i S\xi_i \\ & + \frac{1}{2}k_{i1}(\hat{e}_i^{\times} + \bar{e}_i I_3)(\tilde{\omega}_i - k_{i1}\hat{e}_i)) + \gamma_i(t) + u_i \end{aligned} \quad \text{(21c)}$$

where $\alpha_i(t)$, $\beta_i(t)$ and $\gamma_i(t)$ satisfy

$$\lim_{t\to\infty} \alpha_i(t) = 0, \ \lim_{t\to\infty} \beta_i(t) = 0, \ \lim_{t\to\infty} \gamma_i(t) = 0$$

Simplification

- Objective of Control: $\forall\ i = 1, \ldots, N$, $\lim_{t\to\infty} \hat{e}_i(t) = 0$ and $\lim_{t\to\infty} \tilde{\omega}_i(t) = 0$.
- Lemma 1: Consider (21a) and (21b). If $\tilde{\omega}_i(t)$ is piecewise continuous for $t \geq 0$, and

$$\lim_{t\to\infty} \tilde{\omega}_i(t) = 0,\ i = 1, \ldots, N,$$

then $e_i(t)$ is bounded for all $t \geq 0$ and

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Simplification (Cont.)

- By Lemma 2, it suffices to design a distributed control law of the form (12) to globally stabilize the following error dynamic equation

$$J_i\dot{\tilde{\omega}}_i = -\omega_i^{\times} J_i\omega_i + J_i((\tilde{\omega}_i - k_{i1}\hat{e}_i)^{\times}\hat{C}_i\xi_i - \hat{C}_iS\xi_i + \frac{1}{2}k_{i1}(\hat{e}_i^{\times} + \bar{e}_iI_3)(\tilde{\omega}_i - k_{i1}\hat{e}_i)) + \gamma_i(t) + u_i \quad (22)$$

- To apply the adaptive control technique to system (22), we need to put equation (22) in the standard form where the unknown parameters appear linearly.

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Linear Parameterization

- For any $x = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T \in R^3$, define a linear operator L acting on x by

$$L(x) = \begin{bmatrix} x_1 & 0 & 0 & 0 & x_3 & x_2 \\ 0 & x_2 & 0 & x_3 & 0 & x_1 \\ 0 & 0 & x_3 & x_2 & x_1 & 0 \end{bmatrix}.$$

- Let J_i be denoted by

$$J_i = \begin{bmatrix} J_{i11} & J_{i12} & J_{i13} \\ J_{i12} & J_{i22} & J_{i23} \\ J_{i13} & J_{i23} & J_{i33} \end{bmatrix},$$

and define

$$\Theta_i = \begin{bmatrix} J_{i11} & J_{i22} & J_{i33} & J_{i23} & J_{i13} & J_{i12} \end{bmatrix}^T.$$

Then

$$J_i x = L(x)\Theta_i.$$

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Standard Form

- Thus equation (22) can be rewritten as

$$J_i\dot{\tilde{\omega}}_i = \psi_i(t)\Theta_i + \gamma_i(t) + u_i \tag{23}$$

where

$$\psi_i(t) = -\omega_i^\times L(\omega_i) + L((\tilde{\omega}_i - k_{i1}\hat{e}_i)^\times \hat{C}_i\xi_i - \hat{C}_i S\xi_i + \frac{1}{2}k_{i1}(\hat{e}_i^\times + \bar{e}_i I_3)(\tilde{\omega}_i - k_{i1}\hat{e}_i)).$$

- $\psi_i(t)$ is independent of (q_0, ω_0) whenever $0 \notin \mathcal{N}_i$.

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Distributed Adaptive Control Law

- If $\gamma_i(t)$ is identically zero for all $t \geq 0$, then (23) is in the same form as what was studied in [Chen and Huang 2009] where it was shown that (23) can be globally stabilized by the following adaptive control law:

$$u_i = -\psi_i(t)\hat{\Theta}_i - k_{i2}\tilde{\omega}_i, \quad \dot{\hat{\Theta}}_i = \Lambda_i^{-1}\psi_i(t)^T\tilde{\omega}_i, \quad i = 1, \cdots, N \tag{24}$$

where $k_{i2} > 0$, $\Lambda_i \in R^{6\times 6}$ is some positive definite gain matrix.

- It turns out that, when $\gamma_i(t)$ is not identically zero, but $\lim_{t\to\infty}\gamma_i(t) = 0$, the same control law (24) also globally stabilize (23).
- This control law together with the distributed observer (9) constitutes a distributed adaptive control law of the form (12).

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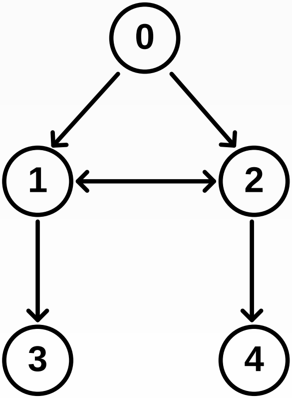
An Example

- Communication network with one leader and four followers:



An Example (Cont.)

- Desirable Angular Velocity Ω_0: Let

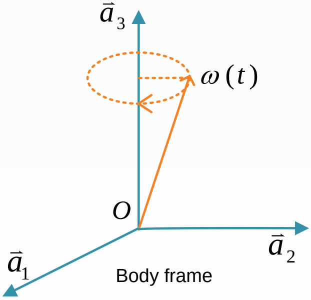
$$\omega_0(t) = [\sin t, \cos t, 3]^T$$

which can be produced by the leader system with

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $\omega_0(0) = [0, 1, 3]^T$

- Initial orientation of the leader: $q_0(0) = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$



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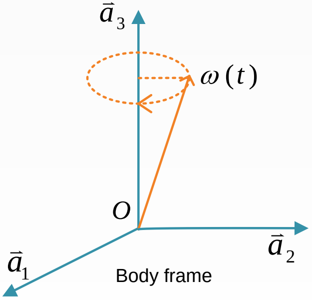

6. Concluding Remarks

- This talk has presented a framework for handling the cooperative control problem of multi-agent systems via the distributed observer approach.
- Under the assumption that the graph is connected, a distributed observer based controller = a decentralized controller + a distributed observer.
- A distributed observer based controller satisfies certainty equivalence principle if it does what a decentralized controller does.

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- Under the assumption that the graph is connected, a distributed observer based controller = a decentralized controller + a distributed observer.
- A distributed observer based controller satisfies certainty equivalence principle if it does what a decentralized controller does.

- The framework has led to a complete solution to the cooperative output regulation problem for general, heterogeneous, uncertain linear multi-agent systems subject to external disturbances.
- It also applies to two classes of practical nonlinear systems, namely, Euler-Lagrange systems and rigid body systems.
- The approach has also been applied to flocking, formation, rendezvous, etc., of some classes of linear systems and Euler-Lagrange systems, thus making the graph connectivity an objective instead of an assumption.
- The cooperative control problem for more general uncertain nonlinear systems can be handled by internal model based approach.

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Thanks

Thank you!

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