

# Distributed control & optimization based on internal model

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**Keywords:** Multi-agent systems (MAS), output regulation (OR), internal model (IM), optimization

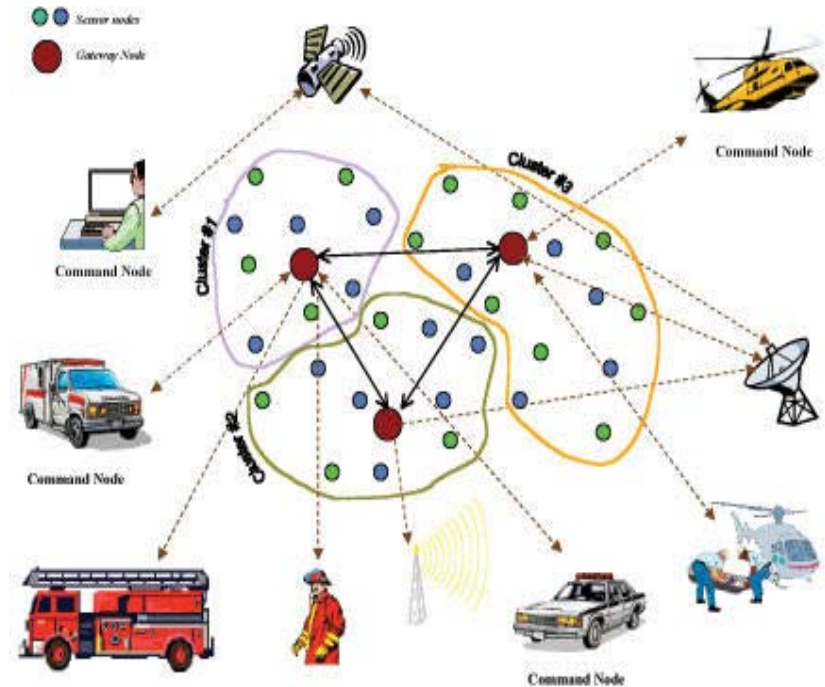
# 内容

- 背景介绍和基本知识
- 分布式输出调节控制
  - Adaptive internal model
  - Networked internal model
- 分布式抗干扰优化
- 结束语

# 一、背景介绍

## Multi-agent systems (多智能体、多自主体、多个体系统)

- **Dynamics:** agent (homogeneous and heterogeneous), environment (passive or active), emergence (split or merge), ...;
- **Information:** directed (agent-agent), indirected (agent-environment), ...;
- **Control:** neighbor-based (switching) rules, mass-based control, partial centralized control, ...



# 多智能体控制和优化问题

- 控制和估计: consensus, containment, formation, flocking, attitude synchronization, Kalman filter, localization , data fusion ...
- 优化和覆盖: distributed optimization, sweep, evasion/pursuit, research/rescue, ...
- 演化和智能算法: opinion dynamics, social networks, evolutionary or swarm intelligence, evolutionary game, ...

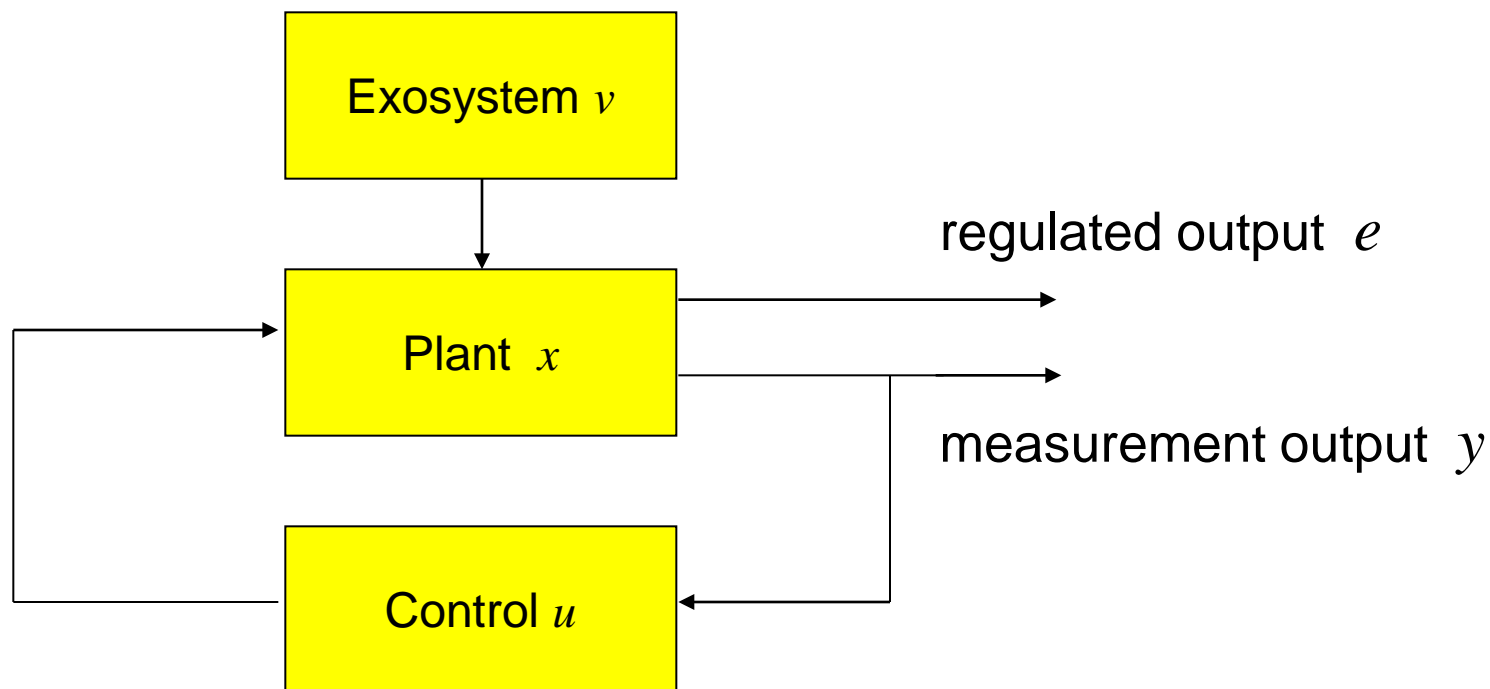
# 我们的一些成果（控制）

- Containment control & multiple leaders (Automatica 2012; IEEE TAC 2012; Automatica 2014): 一个领导者变成一个集合或者多个领导者
- Distributed output regulation (IEEE TAC 2010, 2013, 2014, IJRNC 2013): 领导跟随的一般框架, 包括两个基本方法 (内模方法和观测器方法)
- Attitude synchronization (Automatica 2014): 非线性智能体的动力学; 给出了一般连通性条件下的两类同步算法及其收敛域
- Target surrounding (IEEE TAC 2014): 一群智能体对一个集合的等距离包围
- Distributed Kalman filter (IEEE TAC 2013): 噪声下的分布式估计

# 我们的一些成果（优化等）

- Distributed optimization (IEEE TAC 2013; SCL 2013; IEEE TAC 2014): 随机或确定性下的分布式凸优化
- Coverage: cooperative sweeping (Automatica 2013): 对给定区域带有不确定工作量的清扫覆盖问题
- Quantization in control and optimization (CDC 2013, IEEE TCNS conditionally accepted): 量化下的控制与优化，通讯复杂性
- Opinion dynamics (Physica A 2013): 观点（舆论）的演化分组，特别是其波动性的研究
- ... ..

## 二、分布式输出调节



输出调节 Output regulation:  $e \rightarrow 0$  &  $x$  is bounded.

Stabilization, asymptotical tracking, disturbance rejection ...

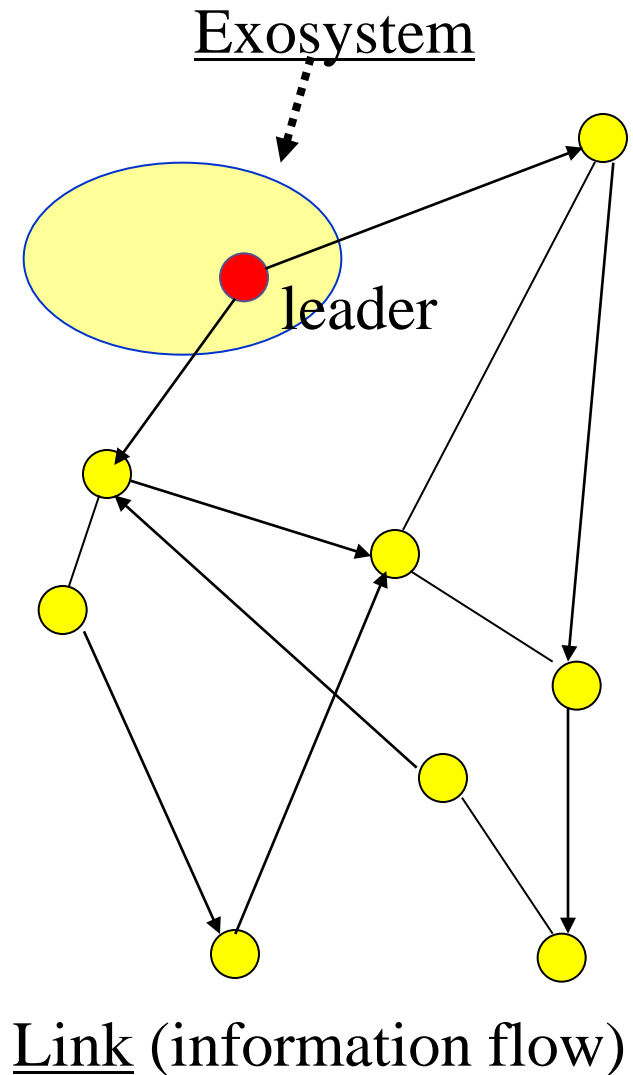


# DOR for leader-following consensus

Motivation of distributed output regulation:

💡 Provide a general framework for leader-following consensus

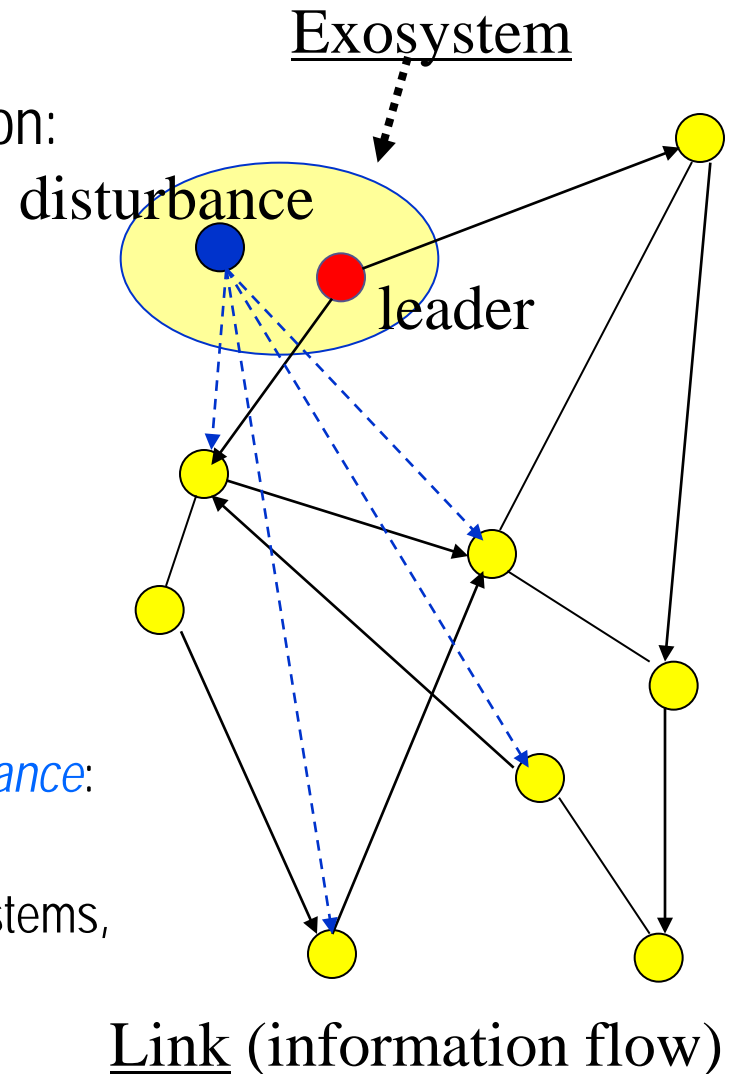
- *Exosystem*: leader (reference signal)
- *Plant*: followers



# DOR beyond leader-following consensus

Motivation of distributed output regulation:

- ✦ Provide a general framework for leader-following consensus
  - *Exosystem*: leader
  - *Plant*: followers
- ✦ Facilitate the study of more coordination problems
  - *Nonlinearity or Uncertainties or Disturbance*: robust and adaptive control ...
  - **Multi-level networks**: cyber-physical systems, combination of decentralized OR and Distributed OR, host internal model ...



# Output regulation (OR)

Two main approaches to output regulation:

- **Observer/estimator (feedforward):** estimate the exosystem and construct regulation feedback
- **Internal model (IM):** build regulation feedback based on IM → robustness ...

# IM for output regulation

- Linear systems (Davison, Wonham, Francis, ...): classic IM → incorporate a model of the exosystem (1970's)
- Nonlinear systems (Isidori, Byrnes, Huang, ...): **Different** IMs → incorporate a model determined jointly by both the **plant** and exosystem (after 1990).
  - Large-scale systems (Ding, Gazi, ...): decentralized OR control → each agent can get the information of the exosystem

# Existing Results

Two main approaches to DOR

- **Distributed observer/estimator:** *Hong et al Automatica 2006; Hong, et al Automatica 2008; Hong et al, JSSC 2009 ...*
- **Distributed internal model:** *Wang, Hong, et al IEEE TAC 2010; Hong et al Int J Robust & Nonlinear Control, 2013; Su, Hong et al IEEE TAC 2013; Xu, Hong et al, IEEE TAC 2014 ...*

# DOR: Fundamental challenges

## Linear systems:

- Solvability: when DOR can be achieved using local neighbor information
- Connectivity condition: fixed or switched?
- Design: how to give IM-based design for DOR

## Nonlinear systems:

- Solvability: necessary/sufficient conditions with given topology?
- Connectivity condition: new communication structure for exchanging local information?
- Design: new IM?

## 2.1 Adaptive DOR

- More complicated dynamics: nonlinear dynamics and uncertain parameters
- Adaptive DOR control for the exosystem with uncertain parameters  $\rightarrow$  nonlinear control

# Nonlinear IM

For nonlinear systems, classic IM does not work → various IMs (canonical IM, etc): incorporate a model determined jointly by both the plant and exosystem

Construct steady-state generator (SSR) for the design of IM



# Steady-state generator (SSG)

SSG is a basic step to construct modern IM for nonlinear systems

For exosystem:  $dv/dt=S(v)$  and  $dx/dt=f(x,u,v,w)$ , if there are smooth functions  $\theta, \alpha, \beta$  vanishing at  $(v,w)=(0,0)$  such that,  $\forall (v,w) \in V \times W$

$$\frac{d\theta(v,w)}{dt} = \alpha(\theta(v,w)), \quad \mathbf{u}(v,w) = \beta(\theta(v,w))$$

where  $\mathbf{u}$  is the solution to the RE .

# From SSG to IM

With SSG  $\{\theta, \alpha, \beta\}$ ,

$$\dot{\eta} = \gamma(\eta, e, u)$$

with output  $u$ , is an IM candidate if  $\forall (v, w)$ ,

$$\alpha(\theta(v, w)) = \gamma(\theta(v, w), 0, \mathbf{u}(v, w)).$$

The IM candidate becomes an IM if it ensures the stabilizability of the closed loop system.

Some known cases: linear SSG (maybe with nonlinear output map), SSG in uniformly observable form...

# Adaptive IM: relatively new

- **Adaptive IMs** for conventional OR for the **exosystem with uncertain parameters**: Serrani, Isidori et al, 2001; Marino & Tomei, 2003; Liu, Huang et al, 2009; Obregon, Castillo et al, 2011
- Su & Huang, SCL 2013: relative degree=1, undirected graph, LH adaptive IM
- Our cases: relative degree= $r > 1$ , directed graph, OC adaptive IM ( $\rightarrow$  containment problem, Automatica, 2014)

# Adaptive DOR

The leader contains uncertain parameters  $\rightarrow$   
DOR design based on adaptive IM to  
make  $e \rightarrow 0$ :

- Leader is linear,  $dv/dt = S(\sigma)v$ , where  $\sigma$  is the uncertain parameter vector
- Followers may be nonlinear

# Design method

Two main design steps: adaptive IM + stabilization of closed-loop systems

- ✦ Adaptive IM: find an observable pair based on the solution of RE (polynomial of  $\nu$ )
- ✦ Stabilization control: construction of Lyapunov function for the closed loop system

The result is consistent with the case when there is no uncertain parameter.

# Results

Adaptive DOR can be solved and the corresponding controller can be constructed for directed graph and minimum phase with relative degree  $r$ .

Design: Construct an adaptive IM; Construct stabilization controller based on the construction of a Lyapunov function for directed graph (it is easier if the graph is undirected for the symmetry of the matrix)

# Example 1

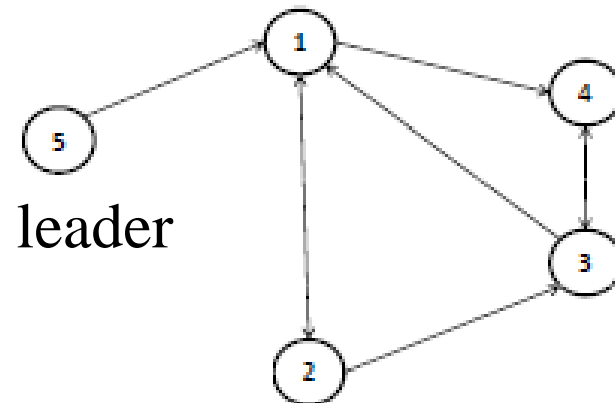
Leader:  
 $\omega$  uncertain

$$\begin{cases} \dot{v}_1 = \omega v_2 \\ \dot{v}_2 = -\omega v_1 \\ y_0 = v_1 \end{cases}$$

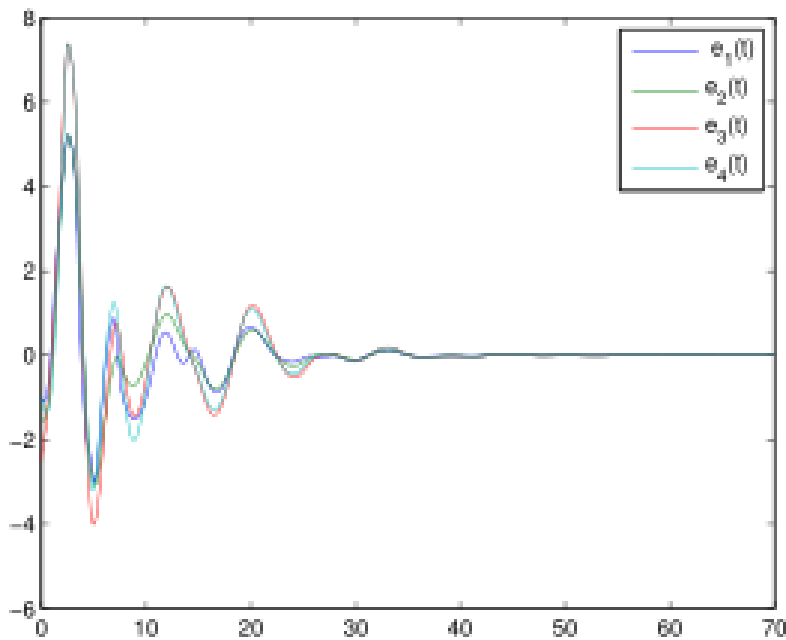
4 followers:

$$\begin{cases} \ddot{y}_i + d_i \dot{y}_i + s_i y_i = u_i \\ 0.5 \leq d_i \leq 1.5, 1 \leq s_i \leq 2 \end{cases}$$

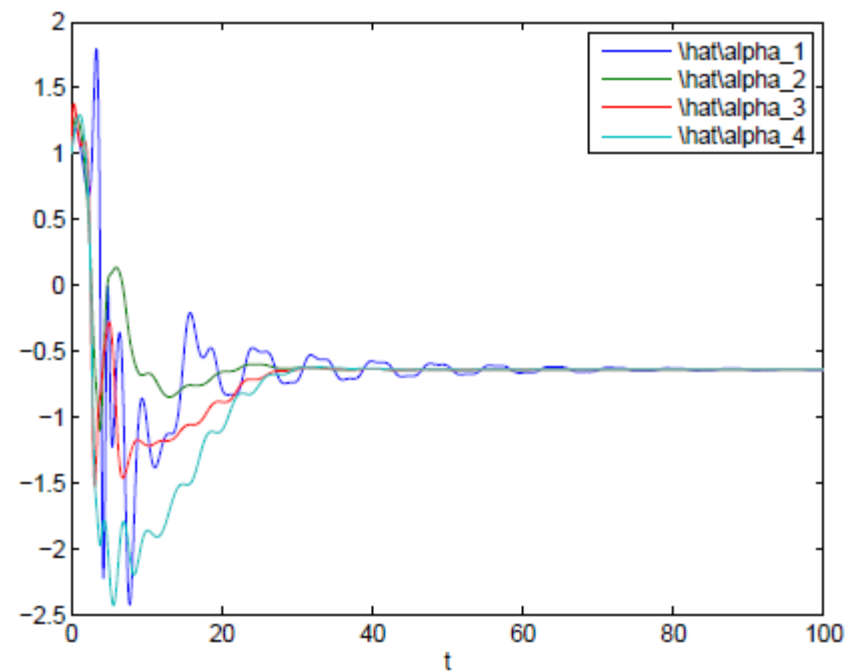
- 4 followers with 1 leader
- Directed graph
- Uncertainties both in leader and followers



# Numerical simulations



Tracking errors of followers:  
Regulated error  $y_i - y_0 \rightarrow 0$



Estimation errors of followers:  
The estimate  $\rightarrow \alpha_0 = -\omega^2$



## 2.2 Networked IM

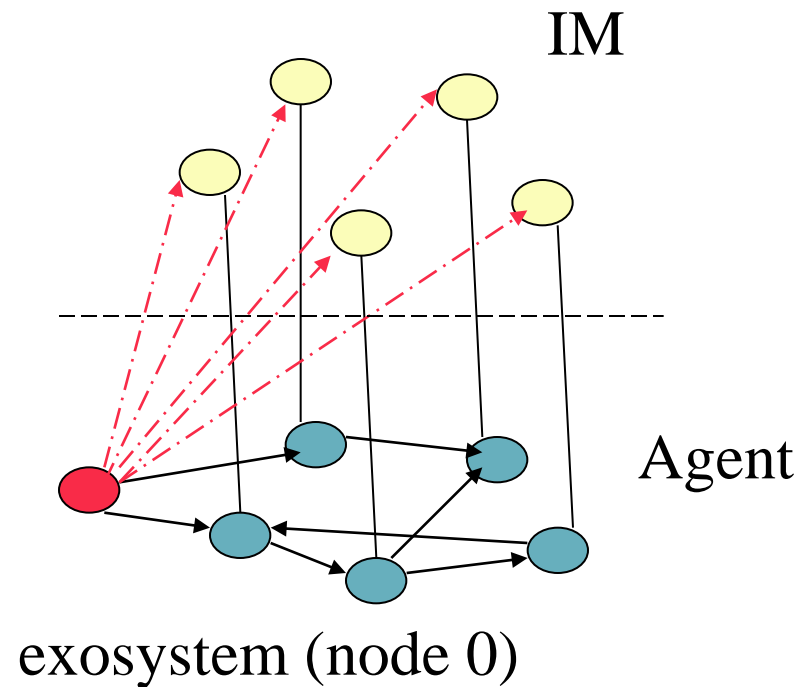
DOR with two-level networks: additional network to exchange IM information (CCC2012, ICARCV 2012, CCC 2013, IEEETAC 2014).

Motivation:

- A framework for distributed OR and decentralized OR (large-scale systems)
- New IM to solve complex network coordination by sharing IM information

# One level graph for DOR

- The graph describe the interaction between agents, but sometimes (nonlinear agents ...), it is hard to achieve DOR only with the graph
- For DOR, each agent has its own IM for the same exosystem (leader or disturbance source)  
→ what a waste?



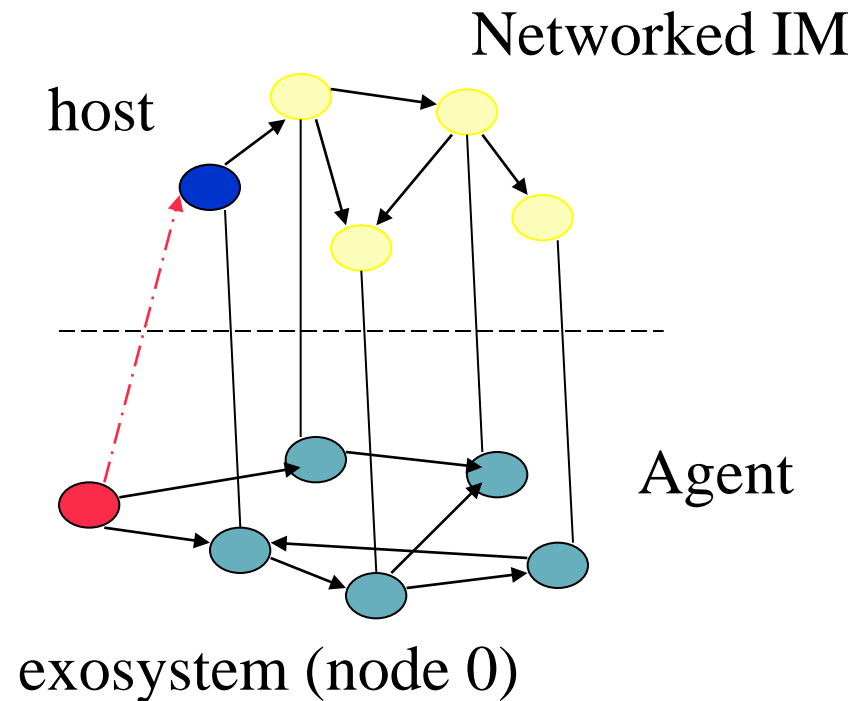
# Two-level graph for DOR

Based on 2-level graph:

1. Plant (agent) graph: physical connection between agents & measurement information
2. IM (controller) graph: communication between controllers

**Effective design for DOR?**

Share IM information,  
improve performance ...



Some agents may not have  
structural information of node 0

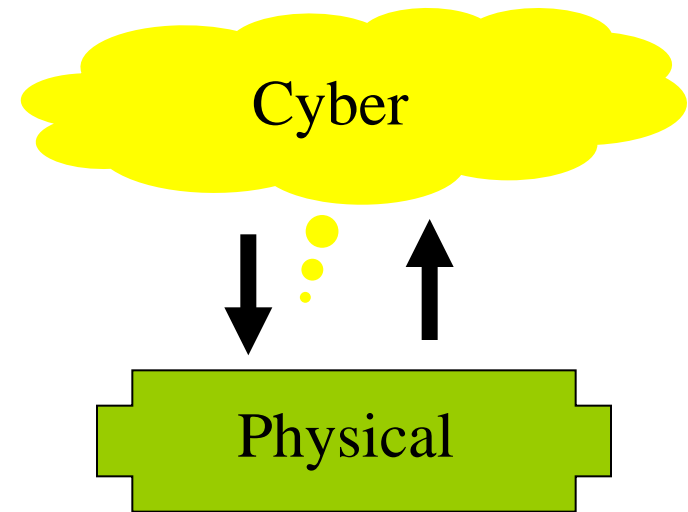
# 2-level network: cyber-physical ?

Unify decentralized and distributed design for OR:

Large-scale decentralized control on physical layer (fixed plant graph)

+

Distributed control on cyber or communication layer (variable controller graph)



Mathematical analysis for 2-level network ...

# Networked IM (CCC2012)

DOR Design to share the information of neighbor IM-based controllers.

IM  $\rightarrow$  networked IM:

$$\dot{\eta}_i = f_i(\eta_i, x_i, \eta^{\mathfrak{N}}) \quad \text{or} \quad \eta_i = g_i(\eta^{\mathfrak{N}})$$

where

$$\eta_i^{\mathfrak{N}} = (\eta_j, j \in \mathcal{O}_i^c)$$

Neighbor information

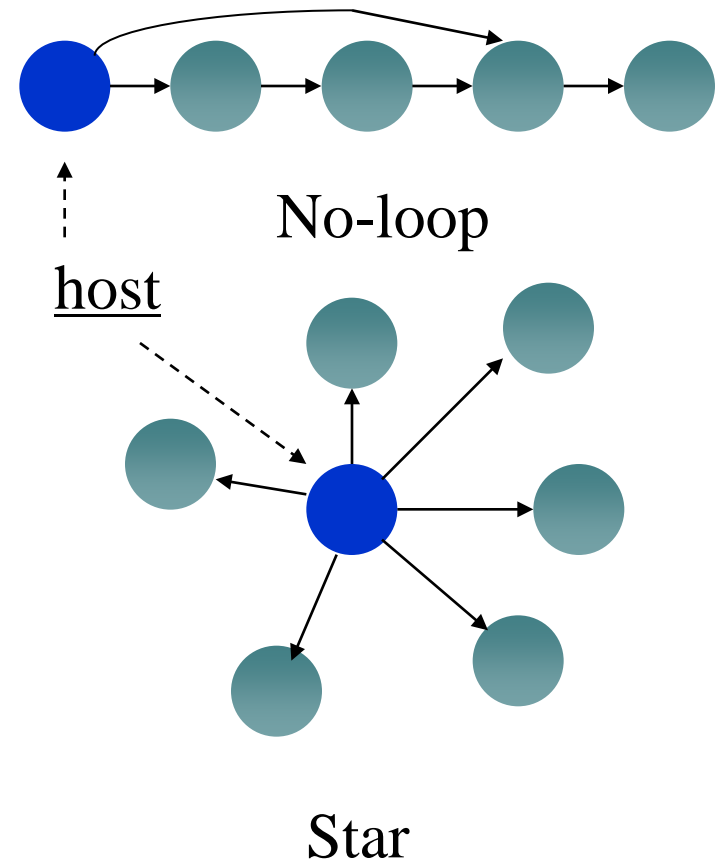
The distributed controller:  $u_i = h_i(e_i, \eta_i)$

Measurement variable

IM variable

# Challenges in IM network design

- Construction of IM
- Structure of IM network (different from the agent network): 2 simple cases
- Selection of host IM for homogenous or heterogeneous agents
- Solution of the RE and construction of Lyapunov function
- Nonlinear and uncertain agents/leaders ...



## Example 2

The exosystem consists of two parts: leader & disturbance

☀ Leader: Lienard system  
 $y_0$  : output of the leader

$$\begin{cases} \dot{v}_{r1} = -G(v_{r2}), \\ \dot{v}_{r2} = v_{r1} - F(v_{r2}) \\ y_0 = v_{r1} \end{cases}$$

☀ Disturbance model:

$$\begin{cases} \dot{v}_{d1} = v_{d2}, \\ \dot{v}_{d2} = -\omega^2 v_{d1} \\ \dot{v}_{d3} = 0 \end{cases}$$

# Heterogeneous followers

Agent 1 with  $x_1$

$$\dot{x}_1 = u_1,$$

$$e_1 = x_1 - y_0$$

Agent 2 with  $x_2$

$$\dot{x}_2 = u_2 + x_1^2 - v_{d1},$$

$$e_2 = x_2 - x_1.$$

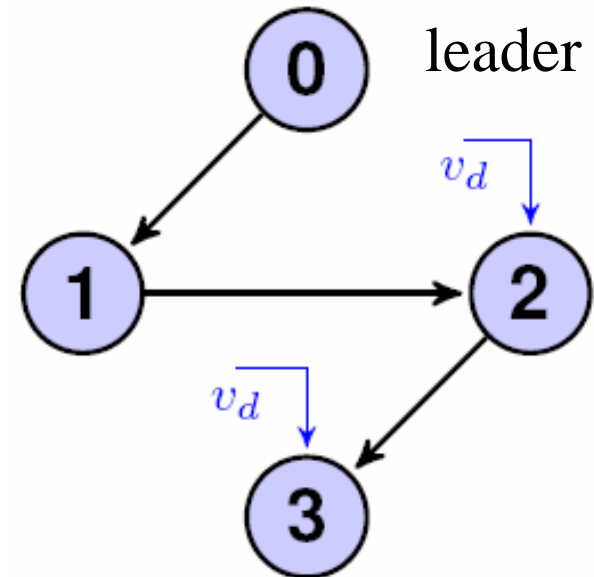
Agent 3 with  $x_3 = \text{col}(z_3 \ y_3)$

$$\dot{z}_3 = -z_3 + e_3 y_3,$$

$$\dot{y}_3 = u_3 - v_{d3} v_{d1} + z_3^2 + v_{d2}^3 - \sin^2(x_2)$$

$$e_3 = y_3 - x_2.$$

Plant/agent graph:

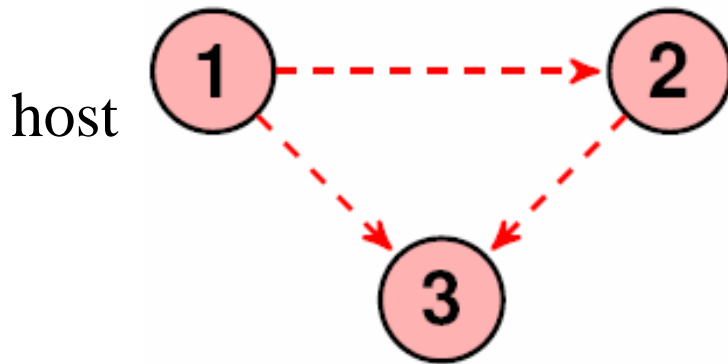


Relative measurement  $e_i$



# IM/Controller graph

IM/controller graph:



Cooperative controller based on networked IM: (cyber) IM variable  $\eta_i$  exchanged over the controller graph

- 💡 Helpful to solve RE
- 💡 Largely reduce structural complexity of IMs for agents 2 and 3,...

# Results (ICARCV 2012)

**With more information exchanges, solve the unsolvable problems in decentralized control, and reduce the complexity of IM-based controller.**

There are solution of RE and SSR for heterogeneous nonlinear agents in output-feedback form → networked IM can be constructed to be input-to-state stable to solve DOR for fixed two-level graphs without loops.

# Results (ICCA2014, IEEE TAC2014...)

## Leader

$$\dot{v}_r = S_r v_r, \quad y_0 = q(v_r, w)$$

## Follower

$$i \in \mathcal{O} : \begin{cases} \dot{z}_i = f(z_i, y_i, w) \\ \dot{y}_i = g(z_i, y_i, w) + \delta_i(v_i) + u_i \end{cases}$$

- $\mathcal{O} := \{1, \dots, N\}$
- **strict-feedback uncertain system having unity relative degree**

## Disturbance

$$i \in \mathcal{O} : \quad \dot{v}_i = S_i v_i$$

# Problem Formulation

## Two outputs

- **Regulated output: control aim, unavailable for control design**

$$i \in \mathcal{O} : e_i = y_i - y_0$$

- **Measurement output: available for control design**

$$i \in \mathcal{O} : e_{mi} = \sum_{j=0}^N a_{ij}(y_i - y_j)$$

# Semi-global leader following

For any index  $j$  and any sets  $\mathcal{B}_\rho^z$  and  $\mathcal{B}_\rho^y$  with

$$z := (z_1, \dots, z_N), \quad y := (y_1, \dots, y_N)$$

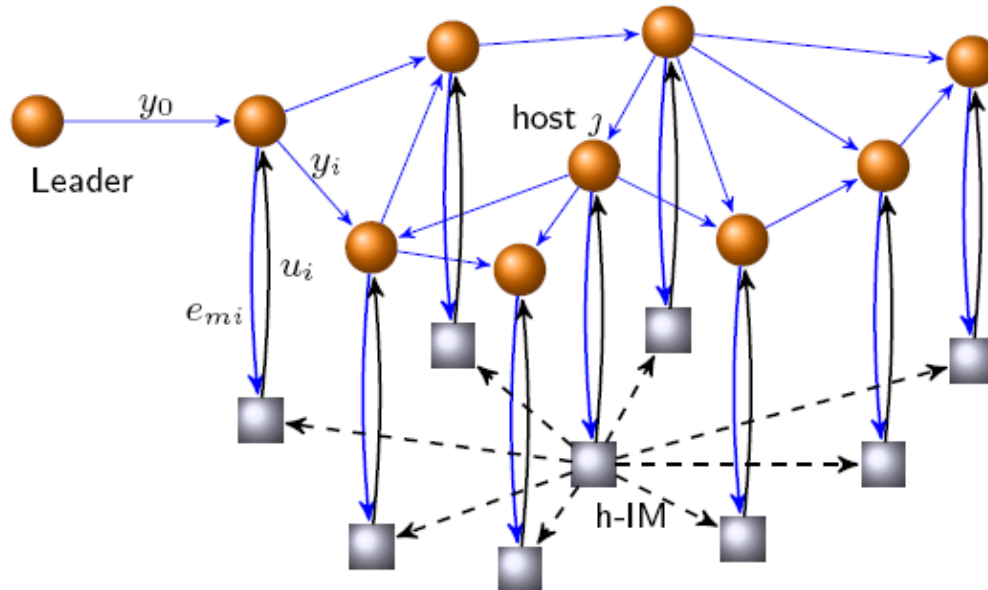
find a smooth distributed controller

$$\begin{cases} \dot{\eta}_j = h_\eta(\eta_j, u_j), & \dot{\xi}_i = h_{\xi_i}(\xi_i, \eta_j, u_i) \\ u_i = u_{ci}(\xi_i, \eta_j, e_{mi}), & i = 1, \dots, N \end{cases}$$

with a set  $\mathcal{B}_{\rho'}^{\xi'}$ , where  $\xi' = (\eta_j, \xi_1, \dots, \xi_N)$ , such that, for any initial conditions in set  $\mathcal{B}' := \mathbb{V} \times \mathbb{W} \times \mathcal{B}_\rho^z \times \mathcal{B}_\rho^y \times \mathcal{B}_{\rho'}^{\xi'}$ ,

1. the trajectory of the closed-loop exists and is bounded;
2.  $\lim_{t \rightarrow \infty} e(t) = 0$

# Host IM



**Host agent: 2 IMs:**

- One IM (host IM) is to track the leader
- local one is to reject local disturbance

**Other agents :** local IM to reject local disturbance

# Main Result

**Theorem:** Under standard assumptions, for any selected host agent index  $j$ , the semi-global leader-following consensus problem can be solved by the host-internal-model-based control

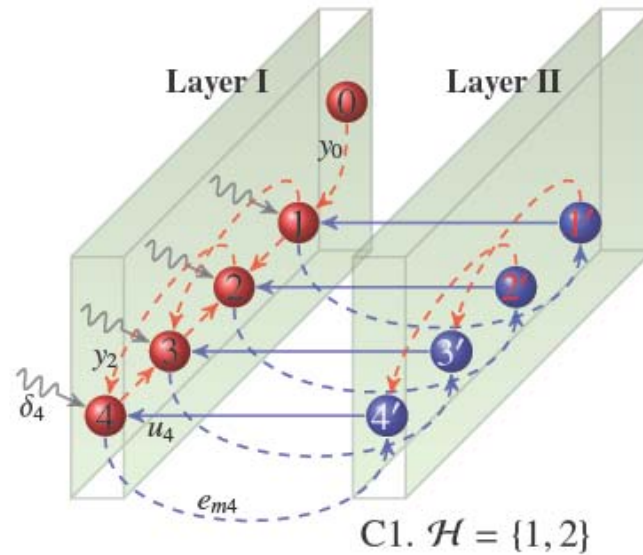
$$i \in \mathcal{O} : \dot{\eta}_i = \begin{cases} M_i \eta_i + \Gamma_i u_i, & i = j \\ M_i \eta_i + \Gamma_i (u_i - \bar{\Psi}_r \eta_j), & i \neq j \end{cases}$$

and stabilization control

$$i \in \mathcal{O} : \bar{u}_i = -k e_{mi}$$

# Remarks

- A striking reduction of the controller order and computing burden, while a one-dimensional signal is transmitted
- More general IM network can also be constructed. For example



C2.  $\mathcal{H} = \{2\}$



C3.  $\mathcal{H} = \{1, 4\}$



C4.  $\mathcal{H} = \{1, 2, 3, 4\}$



# Example 3

Consensus design of a group of FitzHugh-Nagumo type agents with local disturbances

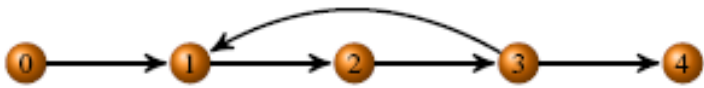
**Leader**  $\dot{v}_{r1} = \omega_r v_{r2}, \dot{v}_{r2} = -\omega_r v_{r1}, y_r = v_{r1}$

**Follower** 
$$\begin{cases} \dot{z}_{i1} = \sigma_5(y_i - \sigma_2 z_{i1}) \\ \dot{z}_{i2} = \sigma_6(-y_i - \sigma_4 z_{i2}) \\ \dot{y}_i = y_i - \frac{1}{3}y_i^3 - z_{i1} + z_{i2} + F_i(t) + u_i, i = 1, \dots, 4 \end{cases}$$

**Disturbance**  $F_i(t) = A_{mi} \sin(\omega_i t + \phi_i) + d_i$

# Simulation

## Measurement graph



## Host agent index

$$j = 2$$

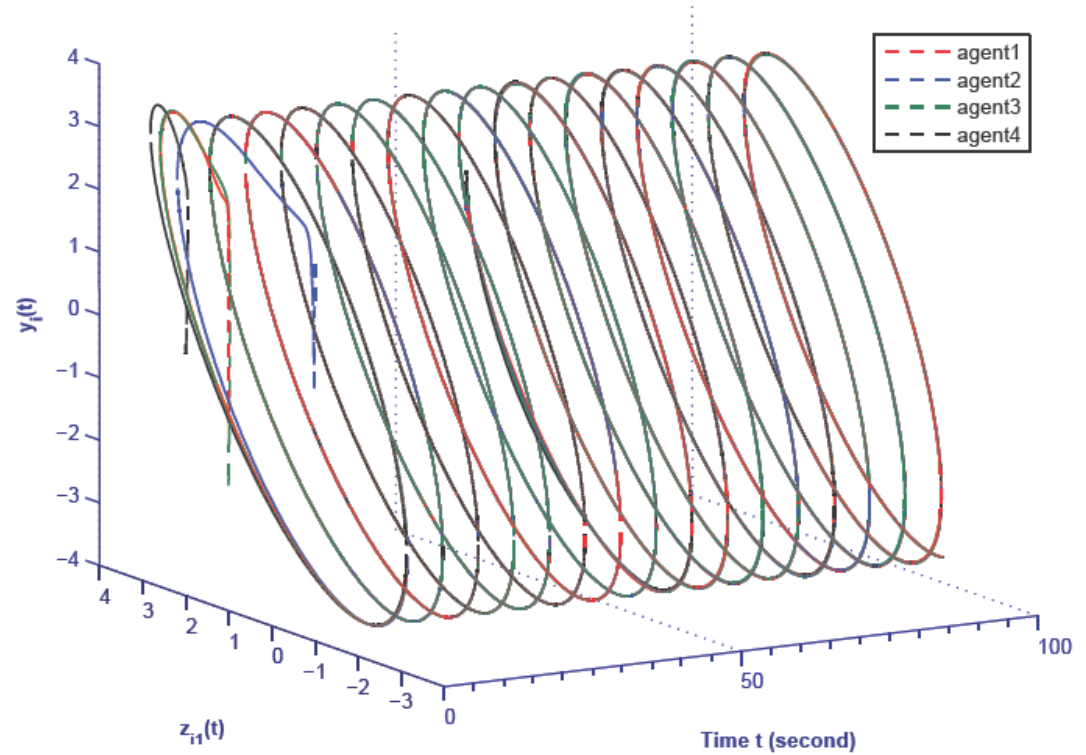


Figure 2: 3D plot of agent responses with axis  $(t, z_{i1}, y_i)$ .

### 三、 分布式抗干扰优化

Convex optimization:  $\min_{z \in R^m} f(z)$

→ Distributed convex optimization:

$$\min_{z \in R^m} f(z) = \sum_{i=1}^n f_i(z)$$

**Extensions:**

- Zero-sum game:  $\min \max f(x, y)$
- Constrained convex optimization → Non-convex optimization

# Basic motivation and idea

- 现有的分布式优化设计没有考虑不确定性对优化过程的影响
- 当要实现分布式优化的个体为现实为物理实体（如机器人、无人车等）时，各种干扰不得不考虑
- 在干扰下实现精确分布式优化：连续时间个体动力学 + 分布式凸优化 + 由一个外在系统产生的干扰

# 基本假设

- 优化函数严格凸和局部**Lipschitz**
- 固定连通图
- 干扰的频率已知

两种两种情况:

- 无向图
- 有向平衡图 + **m-Lipschitz**

# Problem setup

Consider the following system

$$\dot{x}_i = u_i + d_i(t), \quad i = 1, \dots, N$$

where  $x_i$  is the state,  $u_i$  is the input, and  $d_i$  is the disturbance governed by

$$\dot{w}_i = Sw_i, \quad d_i = Cw_i(t)$$

The control aim is to find distributed control

$$\dot{z}_i = g_{i1}(z_i, \nabla f_i(x_i), x_{mi})$$

$$u_i = g_{i2}(z_i, \nabla f_i(x_i), x_{mi})$$

to solve the optimization problem:  $x_i \rightarrow x^*$  with

$$x^* = \arg \min_{x \in \mathbb{R}^n} f(x), \quad -\infty < x^* < +\infty$$

# 抗干扰分布式优化算法

控制器：优化的次梯度项 + 个体协作的趋同项 + 基于内模的抗干扰项。

$$\dot{v}_i = \alpha\beta \sum_{j=1}^N a_{ij}(x_i - x_j)$$

$$\dot{\eta}_i = (I_n \otimes F)\eta_i + (I_n \otimes G)u_i$$

$$u_i = \underbrace{-\alpha \nabla f_i(x_i) - v_i}_{\text{optimal term}} \underbrace{-\beta \sum_{j=1}^N a_{ij}(x_i - x_j)}_{\text{consensus term}} \\ \underbrace{-(I_n \otimes \Psi)\eta_i}_{\text{internal model term}} .$$

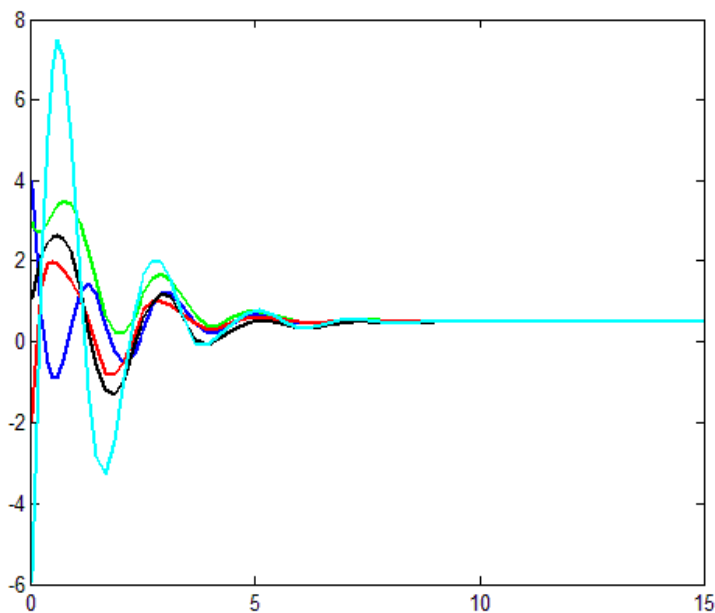
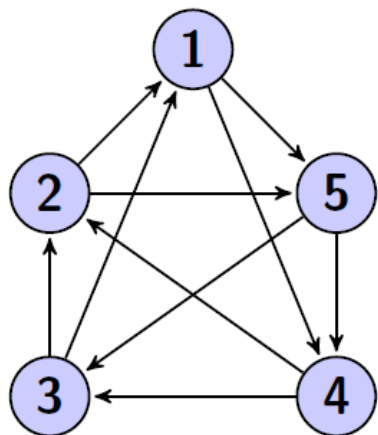
# 主要结果

- 在已有假设下，抗干扰精确优化可以全局实现（**Wang, Yi, Hong, Control Theory & Technology, 2014**）
- 如果干扰频率未知，可采用自适应内模使得抗干扰精确优化半全局实现（**Wang, Hong, Yi, CCC 2014**; 注意到优化函数和未知频率都使得系统变成非线性）
- 结果可以推广到二阶甚至高阶（线性）个体动力学情况(**ongoing work**)



# Example 4 (5 agents)

连通拓扑图和优化误差曲线



$$f_1(x) = (x + 2)^2, \quad f_2(x) = (x - 5)^2$$

$$f_3(x) = x^2 \ln(1 + x^2) + x^2$$

$$f_4(x) = \frac{x^2}{\sqrt{x^2 + 1}} + x^2, \quad f_5(x) = \frac{x^2}{\ln(2 + x^2)}$$

## 4 Conclusions

**MAS: collective dynamics and distributed algorithms**

- **When conventional theory (e.g., output regulation or optimization) meets new topic (e.g., MAS) → Distributed output regulation or optimization: framework, fundamental problems, ...**
- **MAS: many new topics beyond consensus**

**Thank you!**