

Cooperative Control of Multi-agent Systems: A Distributed Observer Approach

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The Chinese University of Hong Kong

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Dynamic Multi-Agent Systems
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Acknowledgement

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Outline

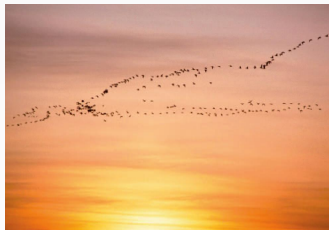
- 1 Introduction
- 2 Classical Output Regulation: Feedforward Control Approach
- 3 Cooperative Control of Multi-Agent Systems
- 4 A Distributed Observer Approach
- 5 A Case Study: : Attitude Consensus of Multiple Spacecraft Systems
- 6 Concluding Remarks

1. Introduction

Collective Behaviors



School of Fish
(S. Martinez, et al. 2007)



Flocking of Birds
(<http://www.fws.gov>)

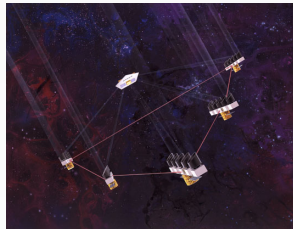


Swarm of Locusts
(<http://sciencephoto.com/image>)

Multi-agent Systems



Robot Formation
 (<http://www-symbiotic.cs.ou.edu>)



Formation of Spacecraft
 (<http://www.acsu.buffalo.edu>)



Flight Formation
 (<http://4.bp.blogspot.com>)

Distributed Control of Multi-Agent Systems (Cont.)

- The individual subsystems can only access the information of their **neighbors**. Thus the system has to be controlled by a **distributed** control protocol featuring the so-called **nearest neighbor rule**.
- Information has to be **shared** among individual agents, and all agents in the group have a **common** objective leading to collective behaviors.
- The global behavior of the system is **jointly** dictated by the system dynamics and the communication topology.
- A basic control problem for multi-agent systems is (leader-following) **consensus**.

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Consensus

- The **leader-following** consensus problem is to design a distributed feedback control law such that the outputs of all agents converge to a **prescribed** trajectory which is usually produced by another agent called **leader**.
- So far, the consensus problem has been mainly studied for **linear**, **homogeneous** multi-agent systems without subjecting to model **uncertainty** and external **disturbances**.
- Other variants of the consensus problem include **synchronization**, **flocking**, **swarming**, **formation**, **rendezvous** (Fax and Murray, 2004), (Jadbabaie, Lin, Morse, 2003), (Ren and Beard, 2008), etc.



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Cooperative Output Regulation

- The output regulation problem aims to deal with the **asymptotic tracking** and **disturbance rejection** problem in an **uncertain** plant. This objective includes the consensus as a special case.
- The cooperative output regulation problem handles the **asymptotic tracking** and **disturbance rejection** problem for **uncertain multi-agent** systems via a **distributed** control scheme. Two approaches, namely, **distributed internal model** based approach and **distributed observer approach** have been developed since 2009.
- An application of the main result will lead to the solution of the leader-following consensus problem for a **nonlinear heterogeneous** multi-agent system subject to model **uncertainty** and external **disturbances**.
- The tools also apply to **flocking, swarming, formation, rendezvous, etc.**

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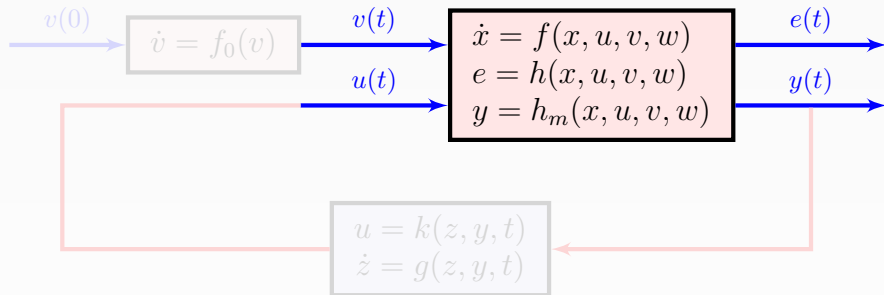
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2. Classical Output Regulation: Feedforward Control Approach

Problem Description

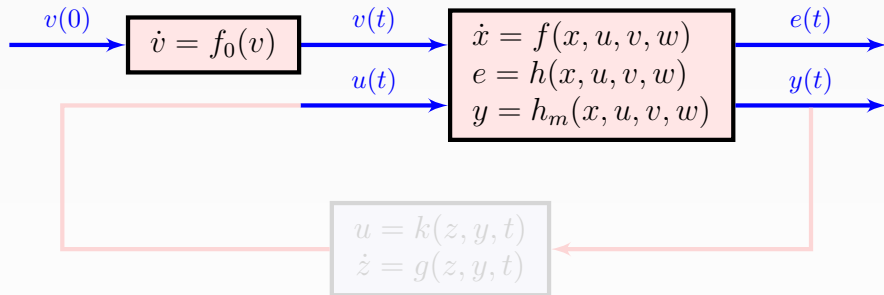


- **Problem Statement:** Design a control law such that, for all $v(t)$, the solution of the closed-loop system is globally bounded, and satisfies

$$\lim_{t \rightarrow \infty} e(t) = 0.$$

- The control law is called **measured output feedback control**. It includes **error output feedback** with $y = e$ and **full information feedback** with $y = (x, v)$ as two special cases.

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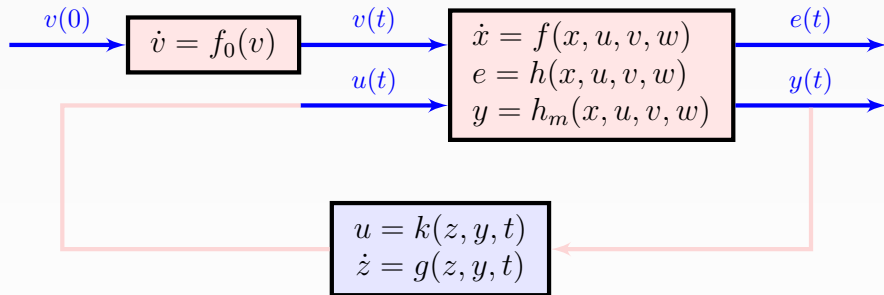


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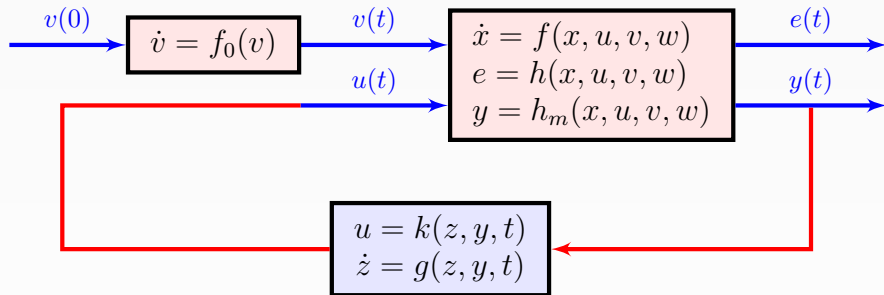


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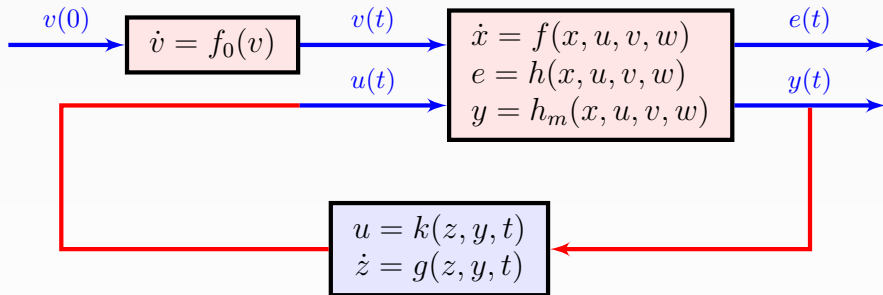


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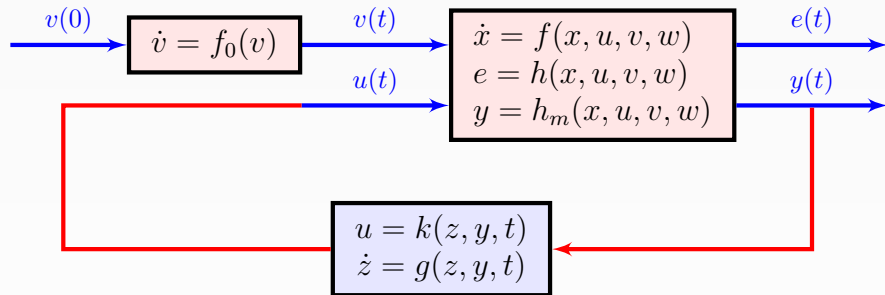


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- The problem handles **asymptotic tracking** and **disturbance rejection** simultaneously where both the reference input and disturbances are generated by the autonomous system $\dot{v} = f_0(v)$ called **exosystem**.
- The problem can be viewed as a **leader-following consensus** problem with the exosystem as the leader and the controlled plant as the single follower.
- Two different approaches, namely, **feedforward** control and **internal model** control have been developed (Isidori, Huang, Khalil, et al).
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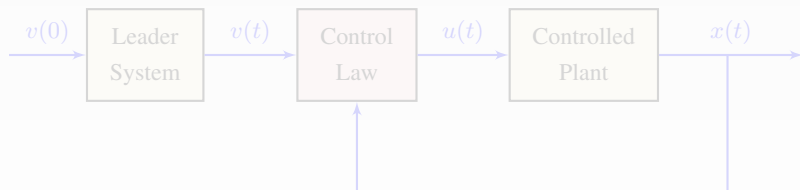
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Feedforward Control Approach

- The feedforward control assumes $y = (x, v)$. The control law is of the following form:

$$u = k(x, v) \quad (1)$$

which leads to the following **closed-loop** system:



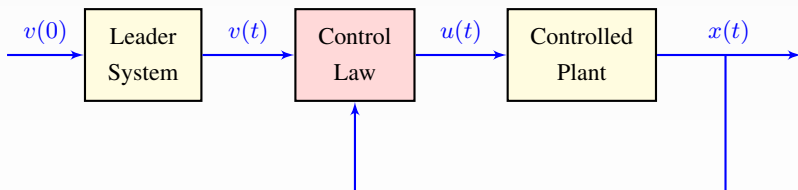
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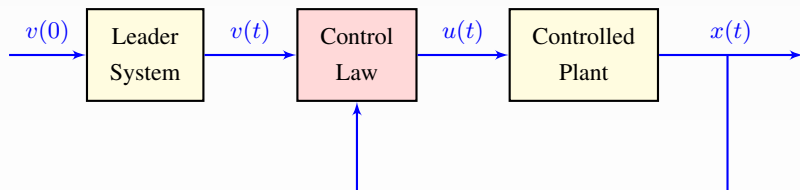
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Observer based Control

- The state $v(t)$ of the exosystem is often **not available** for control. It is desirable to design a control law which only depends on the measured output of the exosystem.
- Given a system of the form

$$\dot{v} = f_0(v), \quad y_o = g_o(v) \quad (2)$$

where y_o is the measured output of (2). The following system

$$\dot{\eta} = \phi(\eta, y_o) \quad (3)$$

is called an **asymptotic observer** of (2) if, for any $v(0)$ and $\eta(0)$,

$$\lim_{t \rightarrow \infty} (\eta(t) - v(t)) = 0 \quad (4)$$

- **Observer based Control Law:**

$$u = k(x, \eta), \quad \dot{\eta} = \phi(\eta, y_o). \quad (5)$$

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Certainty Equivalence Principle

- If an **observer based** control law (5) solves the same problem as the **feedforward** control law (1) does, then this control law is said to satisfy **certainty equivalence principle**.
- For **linear time-invariant systems**, an asymptotic observer exists generically, and the **certainty equivalence principle** always holds.
- For **time-varying systems** or **nonlinear** systems, an asymptotic observer may not exist, and even if it exists, the **certainty equivalence principle** may not hold. This is one of the reasons that makes the control of nonlinear or time-varying systems challenging.

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3. Cooperative Control of Multi-Agent Systems

Multi-agent Systems

$$\begin{aligned}
 \dot{x}_i &= f_i(x_i, u_i, v, w) \\
 e_i &= h_i(x_i, u_i, v, w) \quad , \quad i = 1, \dots, N \\
 y_i &= h_{mi}(x_i, u_i, v, w)
 \end{aligned} \tag{6}$$

where $x_i \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{m_i}$, $e_i \in \mathbb{R}^p$, and $y_i \in \mathbb{R}^{p_i}$. The exogenous signal $v \in \mathbb{R}^q$ is generated by the following exosystem:

$$\dot{v} = f_0(v), \quad y_0 = h_0(v) \tag{7}$$

- System (6) together with (7) can be viewed as a **multi-agent** system of $N + 1$ agents where the exosystem (7) is viewed as the **leader**, and all subsystems of system (6) are viewed as N **followers**.
- If all followers can access the state v of the leader, then the output regulation problem of (6) and (7) can be handled by a so-called **decentralized** control scheme.

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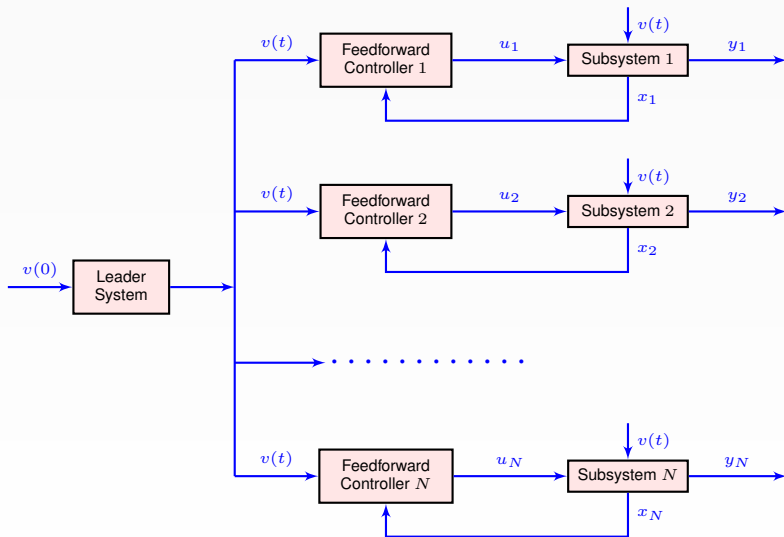
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Decentralized Feedforward Control





Communication Graph

- Given the multi-agent system (6) and (7), one can define a communication graph $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$ with \mathcal{V} being the node set and $\mathcal{E}(t)$ being the edge set.
- $\mathcal{V} = \{0, 1, \dots, N\}$ with the node 0 associated with (7) and the other N nodes associated with the N followers of (6).
- For any $t \geq 0$, $\mathcal{E}(t) \subset \mathcal{V} \times \mathcal{V}$. $(j, i) \in \mathcal{E}(t)$, $i \neq j$, $i, j = 0, 1, \dots, N$, if and only if the control u_i of the subsystem i , $i = 1, \dots, N$, can access y_j at time t , $j = 0, 1, \dots, N$. j is said to be a neighbor of i at time t .
- $\mathcal{N}_i(t) = \{j, (j, i) \in \mathcal{E}(t)\}$ denotes the neighbor set of the node i at time t .
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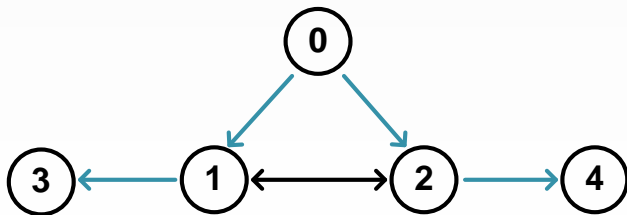
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- $\mathcal{N}_i(t) = \{j, (j, i) \in \mathcal{E}(t)\}$ denotes the neighbor set of the node i at time t .
- $\mathcal{G}(t)$ is said to be static if $\mathcal{G}(t) = \mathcal{G}(0)$ for any $t \geq 0$.

Communication Graph

- Given the multi-agent system (6) and (7), one can define a communication graph $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$ with \mathcal{V} being the node set and $\mathcal{E}(t)$ being the edge set.
- $\mathcal{V} = \{0, 1, \dots, N\}$ with the node 0 associated with (7) and the other N nodes associated with the N followers of (6).
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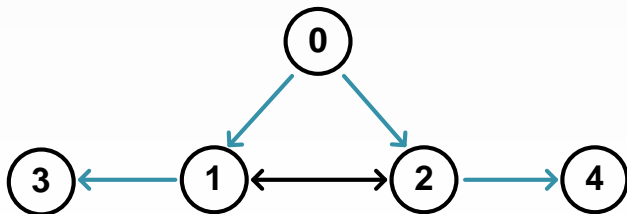
Graph (Cont.)

- A subset of $\mathcal{E}(t)$ of the form $\{(i_1, i_2), (i_2, i_3), \dots, (i_{k-1}, i_k)\}$ is called a **path** of $\mathcal{G}(t)$ from i_1 to i_k at time t , and it is said that the node i_1 can **reach** the node i_k at time t .
- If, at some t , the node 0 can reach all other nodes, then the graph $\mathcal{G}(t)$ is said to be **connected** at time t .



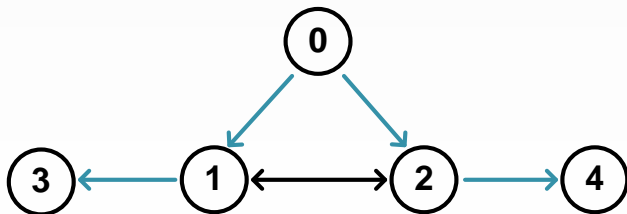
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Distributed Control Laws

- Distributed control law:

$$\begin{aligned} u_i &= k_i(z_i, y_i, y_j, j \in \mathcal{N}_i(t)) \\ \dot{z}_i &= g_i(z_i, y_i, y_j, j \in \mathcal{N}_i(t)), \quad i = 1, \dots, N \end{aligned} \quad (8)$$

where $y_0 = h_0(v)$, k_i and g_i are some sufficiently smooth functions.

- Control law (8) satisfies the communication constraints: the i^{th} control u_i depends on y_j iff the agent j is a **neighbor** of the agent i .

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□ Problem Formulation

➤ **Definition:** Given the **plant**, the **exosystem**, and the **graph** $\mathcal{G}(t)$, find a distributed control law such that, for any initial condition, the solution of the closed-loop system is bounded, and the error output satisfies

$$\lim_{t \rightarrow \infty} e_i(t) = 0, \quad i = 1, \dots, N.$$

➤ **Remark 1:** The degree of the difficulty of the problem not only depends on the dynamics of the system, but also the property of the graph $\mathcal{G}(t)$ which can be **static** or **time-varying** satisfying such conditions as **every time connected**, **frequently connected**, or **jointly connected**.

➤ **Remark 2:** The **jointly connected** condition is the **most challenging** one since such a graph is **discontinuous** and can be **disconnected** at any time instant.

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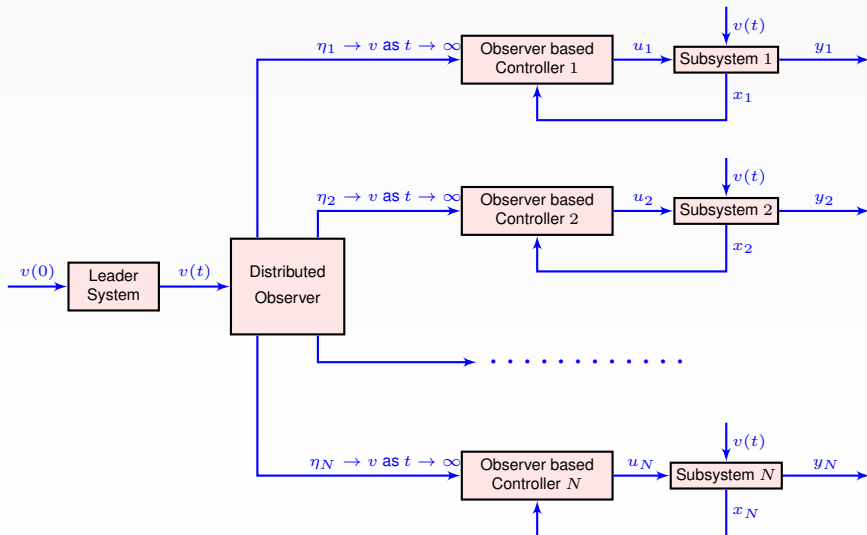
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4. A Distributed Observer Approach

A Distributed Observer Based Scheme





Two Technical Issues

- Does such a distributed observer **exist**?
- Does the **certainty equivalence principle** hold?



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Distributed Observer Candidate

- Given the leader system $\dot{v} = f_0(v)$, $y_0 = h_0(v)$ and a graph $\mathcal{G}(t)$ with $N + 1$ nodes, for $i = 1, \dots, N$, $j = 0, 1, \dots, N$, let $a_{ij}(t) > 0$ if $j \in \mathcal{N}_i(t)$, and $a_{ij}(t) = 0$ if otherwise. Then the following **compensator**

$$\dot{\eta}_i = f_0(\eta_i) + \mu \left(\sum_{j \in \mathcal{N}_i(t)} a_{ij}(t)(\eta_j - \eta_i) \right), \quad i = 1, \dots, N \quad (9)$$

where $\mu > 0$, $\eta_0 = y_0$, is called a **distributed observer candidate** of the leader system, and is called a **distributed observer** of the leader if

$$\lim_{t \rightarrow \infty} (\eta_i(t) - v(t)) = 0, \quad i = 1, \dots, N \quad (10)$$

- Whether or not (9) is a **distributed observer** depends on both the dynamics of the leader and the property of the graph.

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□ Results on Linear Multi-agent Systems

➤ **Proposition 1:** Suppose the leader system is linear, and marginally stable, and the graph $\mathcal{G}(t)$ is **jointly connected**. Then there exists positive μ such that, for all $v(0), \eta_i(0), i = 1, \dots, N$, the solution of (9) satisfies

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Thus, (9) is a distributed **observer** of the leader.

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$$M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, \quad i = 1, \dots, N \quad (13)$$

where $q_i, \dot{q}_i \in R^n$ are the generalized position and velocity vectors, and $\tau_i \in R^n$ is the control input.

➤ Leader System:

$$\dot{v} = Sv, \quad q_0 = Fv \quad (14)$$

where $v \in R^m$, $q_0 \in R^n$, $S \in R^{m \times m}$ and $F \in R^{n \times m}$.

➤ **Remark 4:** System (14) can generate a large class of leader signals such as **step function** of arbitrary magnitude, **ramp function** of arbitrary slope, and **sinusoidal function** of arbitrary amplitude and initial phase.

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- **Theorem 2:** Suppose the system matrix S of the leader system is marginally stable, and the graph $\mathcal{G}(t)$ is **jointly connected**. Then the **certainty equivalence principle** holds for multiple EL systems.
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➤ Multiple Rigid Body Systems (Sidi 1997):

$$\dot{\hat{q}}_i = \frac{1}{2}\hat{q}_i^\times \omega_i + \frac{1}{2}\bar{q}_i \omega_i, \quad \dot{\bar{q}}_i = -\frac{1}{2}\hat{q}_i^T \omega_i \quad (15a)$$

$$J_i \dot{\omega}_i = -\omega_i^\times J_i \omega_i + u_i, \quad i = 1, \dots, N \quad (15b)$$

where $\hat{q}_i \in R^3$ and $\bar{q}_i \in R$, and $q_i = [\hat{q}_i, \bar{q}_i]^T$ is the unit quaternion representing the **attitude** of the i^{th} rigid body, $\omega_i \in R^3$ is the **angular velocity** of the i^{th} rigid body, and $J_i \in R^{3 \times 3}$ and $u_i \in R^3$ denote the inertia matrix and the control torque of the i^{th} rigid body, respectively.

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$$\dot{\omega}_0 = S \omega_0$$

where S is some marginally stable matrix.

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- **Theorem 3:** Given a static graph \mathcal{G} , the **certainty equivalence principle** holds for the multiple rigid body system iff the graph \mathcal{G} is connected.
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5. A Case Study: Attitude Consensus of Multiple Spacecraft Systems

Certainty Equivalence Controller

- True Tracking Error:

$$\epsilon_i = q_0^{-1} \odot q_i$$

$$\hat{\omega}_i = \omega_i - C_i \omega_0$$

The error signals ϵ_i and $\hat{\omega}_i$ are **not available** for every follower.

- Estimated Tracking Error: Partition $\eta_i = [\zeta_i^T, \xi^T]^T$ with $\alpha_i \in R^4$.
Then

$$e_i = \zeta_i^{-1} \odot q_i \tag{18a}$$

$$\bar{\omega}_i = \omega_i - \hat{C}_i \xi_i \tag{18b}$$

where $\hat{C}_i = (\bar{e}_i^2 - \hat{e}_i^T \hat{e}_i) I_3 + 2\hat{e}_i \hat{e}_i^T - 2\bar{e}_i \hat{e}_i^\times$

- **Theorem 3** implies that, $\forall i = 1, \dots, N$,

$$\lim_{t \rightarrow \infty} (e_i(t) - \epsilon_i(t)) = 0, \quad \lim_{t \rightarrow \infty} (\bar{\omega}_i(t) - \hat{\omega}_i(t)) = 0$$

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where $\hat{C}_i = (\bar{e}_i^2 - \hat{e}_i^T \hat{e}_i) I_3 + 2\hat{e}_i \hat{e}_i^T - 2\bar{e}_i \hat{e}_i^\times$

- **Theorem 3** implies that, $\forall i = 1, \dots, N$,

$$\lim_{t \rightarrow \infty} (e_i(t) - \epsilon_i(t)) = 0, \quad \lim_{t \rightarrow \infty} (\bar{\omega}_i(t) - \hat{\omega}_i(t)) = 0$$

Certainty Equivalence Controller

- True Tracking Error:

$$\epsilon_i = q_0^{-1} \odot q_i$$

$$\hat{\omega}_i = \omega_i - C_i \omega_0$$

The error signals ϵ_i and $\hat{\omega}_i$ are **not available** for every follower.

- **Estimated Tracking Error:** Partition $\eta_i = [\zeta_i^T, \xi^T]$ with $\alpha_i \in R^4$.
Then

$$e_i = \zeta_i^{-1} \odot q_i \quad (18a)$$

$$\bar{\omega}_i = \omega_i - \hat{C}_i \xi_i \quad (18b)$$

where $\hat{C}_i = (\bar{e}_i^2 - \hat{e}_i^T \hat{e}_i) I_3 + 2\hat{e}_i \hat{e}_i^T - 2\bar{e}_i \hat{e}_i^\times$

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□ Certainty Equivalence Controller

- Decentralized Controller:

$$u_i = \omega_i^\times J_i \omega_i - J_i (\hat{\omega}_i^\times C_i \omega_i - C_i S \omega_i) - k_{1i} \hat{e}_i - k_{2i} \hat{\omega}_i, \quad i = 1, 2, \dots, N$$

where $k_{1i}, k_{2i} > 0$.

- Certainty Equivalence Controller:

$$u_i = \omega_i^\times J_i \omega_i - J_i (\bar{\omega}_i^\times \hat{C}_i \xi_i - \hat{C}_i S \xi_i) - k_{1i} \hat{e}_i - k_{2i} \bar{\omega}_i \quad (19)$$

$$\dot{\eta}_i = f_0(\eta_i) + \mu \left(\sum_{j \in \mathcal{N}_i(t)} a_{ij}(t) (\eta_j - \eta_i) \right), \quad i = 1, \dots, N \quad (20)$$

- J_i is **uncertain** due to
- uncertain mass distribution;
 - fuel consumption;
 - spacecraft reconfiguration;

Therefore, the control law u_i must not rely on J_i . **Adaptive control** is such a control scheme.

The Error System

- To simplify the closed-loop system analysis, performing on (18a) the following **transformation**

$$\tilde{\omega}_i = \bar{\omega}_i + k_{i1} \hat{e}_i$$

where $k_{i1} > 0$, leads to the following **error system**:

$$\dot{\hat{e}}_i = \frac{1}{2}(\hat{e}_i^\times + \bar{e}_i I_3)(\tilde{\omega}_i - k_{i1} \hat{e}_i) + \alpha_i(t) \quad (21a)$$

$$\dot{\hat{e}}_i = -\frac{1}{2}\hat{e}_i^T(\tilde{\omega}_i - k_{i1} \hat{e}_i) + \beta_i(t) \quad (21b)$$

$$\begin{aligned} J_i \dot{\tilde{\omega}}_i = & -\omega_i^\times J_i \omega_i + J_i((\tilde{\omega}_i - k_{i1} \hat{e}_i)^\times \hat{C}_i \xi_i - \hat{C}_i S \xi_i \\ & + \frac{1}{2}k_{i1}(\hat{e}_i^\times + \bar{e}_i I_3)(\tilde{\omega}_i - k_{i1} \hat{e}_i)) + \gamma_i(t) + u_i \end{aligned} \quad (21c)$$

where $\alpha_i(t)$, $\beta_i(t)$ and $\gamma_i(t)$ satisfy

$$\lim_{t \rightarrow \infty} \alpha_i(t) = 0, \quad \lim_{t \rightarrow \infty} \beta_i(t) = 0, \quad \lim_{t \rightarrow \infty} \gamma_i(t) = 0$$

Simplification

- **Objective of Control:** $\forall i = 1, \dots, N, \lim_{t \rightarrow \infty} \hat{e}_i(t) = 0$ and $\lim_{t \rightarrow \infty} \tilde{\omega}_i(t) = 0$.
- **Lemma 1:** Consider (21a) and (21b). If $\tilde{\omega}_i(t)$ is piecewise continuous for $t \geq 0$, and

$$\lim_{t \rightarrow \infty} \tilde{\omega}_i(t) = 0, \quad i = 1, \dots, N,$$

then $e_i(t)$ is bounded for all $t \geq 0$ and

$$\lim_{t \rightarrow \infty} \hat{e}_i(t) = 0, \quad i = 1, \dots, N.$$

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□ Simplification (Cont.)

- By **Lemma 2**, it suffices to design a distributed control law of the form (12) to globally stabilize the following error dynamic equation

$$\begin{aligned}
 J_i \dot{\tilde{\omega}}_i &= -\omega_i^\times J_i \omega_i + J_i ((\tilde{\omega}_i - k_{i1} \hat{e}_i)^\times \hat{C}_i \xi_i - \hat{C}_i S \xi_i \\
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- To apply the adaptive control technique to system (22), we need to put equation (22) in the standard form where the unknown parameters **appear linearly**.

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Linear Parameterization

- For any $x = [x_1 \ x_2 \ x_3]^T \in R^3$, define a linear operator L acting on x by

$$L(x) = \begin{bmatrix} x_1 & 0 & 0 & 0 & x_3 & x_2 \\ 0 & x_2 & 0 & x_3 & 0 & x_1 \\ 0 & 0 & x_3 & x_2 & x_1 & 0 \end{bmatrix}.$$

- Let J_i be denoted by

$$J_i = \begin{bmatrix} J_{i11} & J_{i12} & J_{i13} \\ J_{i12} & J_{i22} & J_{i23} \\ J_{i13} & J_{i23} & J_{i33} \end{bmatrix},$$

and define

$$\Theta_i = [J_{i11} \ J_{i22} \ J_{i33} \ J_{i23} \ J_{i13} \ J_{i12}]^T.$$

Then

$$J_i x = L(x) \Theta_i.$$

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Standard Form

- Thus equation (22) can be rewritten as

$$J_i \dot{\tilde{\omega}}_i = \psi_i(t) \Theta_i + \gamma_i(t) + u_i \quad (23)$$

where

$$\begin{aligned} \psi_i(t) = & -\omega_i^\times L(\omega_i) + L((\tilde{\omega}_i - k_{i1} \hat{e}_i)^\times \hat{C}_i \xi_i - \hat{C}_i S \xi_i \\ & + \frac{1}{2} k_{i1} (\hat{e}_i^\times + \bar{e}_i I_3) (\tilde{\omega}_i - k_{i1} \hat{e}_i)). \end{aligned}$$

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Distributed Adaptive Control Law

- If $\gamma_i(t)$ is identically zero for all $t \geq 0$, then (23) is in the same form as what was studied in [Chen and Huang 2009] where it was shown that (23) can be globally stabilized by the following adaptive control law:

$$u_i = -\psi_i(t)\hat{\Theta}_i - k_{i2}\tilde{\omega}_i, \quad \dot{\hat{\Theta}}_i = \Lambda_i^{-1}\psi_i(t)^T\tilde{\omega}_i, \quad i = 1, \dots, N \quad (24)$$

where $k_{i2} > 0$, $\Lambda_i \in R^{6 \times 6}$ is some positive definite gain matrix.

- It turns out that, when $\gamma_i(t)$ is not identically zero, but $\lim_{t \rightarrow \infty} \gamma_i(t) = 0$, the same control law (24) also globally stabilize (23).
- This control law together with the distributed observer (9) constitutes a **distributed adaptive** control law of the form (12).

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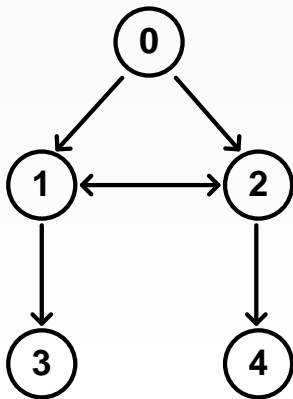
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An Example

- **Communication network** with one leader and four followers:



An Example (Cont.)

- Desirable Angular Velocity Ω_0 : Let

$$\omega_0(t) = [\sin t, \cos t, 3]^T$$

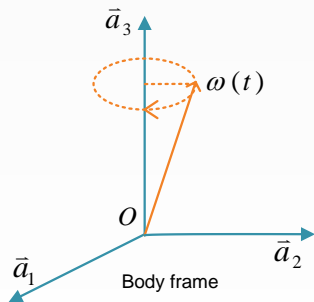
which can be produced by the leader system with

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $\omega_0(0) = [0, 1, 3]^T$

- Initial orientation of the leader:

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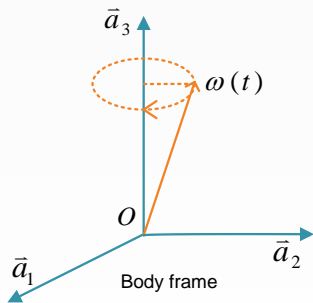
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6. Concluding Remarks

- This talk has presented a framework for handling the cooperative control problem of multi-agent systems via the **distributed observer approach**.
- Under the assumption that the graph is connected, a distributed **observer** based controller = a **decentralized** controller + a **distributed observer**.
- A distributed **observer based controller** satisfies **certainty equivalence principle** if it does what a decentralized controller does.

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- The framework has led to a complete solution to the cooperative output regulation problem for **general, heterogeneous, uncertain** linear multi-agent systems subject to **external disturbances**.
- It also applies to two classes of practical **nonlinear** systems, namely, **Euler-Lagrange** systems and **rigid body** systems.
- The approach has also been applied to **flocking, formation, rendezvous**, etc., of some classes of **linear systems** and **Euler-Lagrange** systems, thus making the **graph connectivity** an objective instead of an assumption.
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Thanks

Thank you!

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