Cooperative Control of Multi-agent Systems: A Distributed Observer Approach

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Acknowledgement

The presentation is based on a joint research with my PhD students Youfeng Su and He Cai.

The research was supported by grants from the Research Grants Council of the Hong Kong Special Administration Region (RGC Ref. No. 412612 and 412813).
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Outline

1. Introduction
2. Classical Output Regulation: Feedforward Control Approach
3. Cooperative Control of Multi-Agent Systems
4. A Distributed Observer Approach
5. A Case Study: Attitude Consensus of Multiple Spacecraft Systems
6. Concluding Remarks
1. Introduction
Collective Behaviors

School of Fish (S. Martinez, et al. 2007)

Flocking of Birds (http://www.fws.gov)

Swarm of Locusts (http://sciencephoto.com/image)
Multi-agent Systems

Robot Formation
(http://www-symbiotic.cs.ou.edu)

Formation of Spacecraft
(http://www.acsu.buffalo.edu)

Flight Formation
(http://4.bp.blogspot.com)
The individual subsystems can only access the information of their neighbors. Thus the system has to be controlled by a distributed control protocol featuring the so-called nearest neighbor rule.

Information has to be shared among individual agents, and all agents in the group have a common objective leading to collective behaviors.

The global behavior of the system is jointly dictated by the system dynamics and the communication topology.

A basic control problem for multi-agent systems is (leader-following) consensus.
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Consensus

- The leader-following consensus problem is to design a distributed feedback control law such that the outputs of all agents converge to a prescribed trajectory which is usually produced by another agent called leader.

- So far, the consensus problem has been mainly studied for linear, homogeneous multi-agent systems without subjecting to model uncertainty and external disturbances.

- Other variants of the consensus problem include synchronization, flocking, swarming, formation, rendezvous (Fax and Murray, 2004), (Jadbabaie, Lin, Morse, 2003), (Ren and Beard, 2008), etc.
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Cooperative Output Regulation

The output regulation problem aims to deal with the asymptotic tracking and disturbance rejection problem in an uncertain plant. This objective includes the consensus as a special case.

The cooperative output regulation problem handles the asymptotic tracking and disturbance rejection problem for uncertain multi-agent systems via a distributed control scheme. Two approaches, namely, distributed internal model based approach and distributed observer approach have been developed since 2009.

An application of the main result will lead to the solution of the leader-following consensus problem for a nonlinear heterogeneous multi-agent system subject to model uncertainty and external disturbances.

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2. Classical Output Regulation: Feedforward Control Approach
Problem Description

\[ \dot{x} = f(x, u, v, w) \]
\[ e = h(x, u, v, w) \]
\[ y = h_m(x, u, v, w) \]
\[ \dot{v} = f_0(v) \]
\[ u = k(z, y, t) \]
\[ \dot{z} = g(z, y, t) \]

Problem Statement: Design a control law such that, for all \( v(t) \), the solution of the closed-loop system is globally bounded, and satisfies

\[ \lim_{t \to \infty} e(t) = 0. \]

The control law is called measured output feedback control. It includes error output feedback with \( y = e \) and full information feedback with \( y = (x, v) \) as two special cases.
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Problem Description (Cont.)

- The problem handles asymptotic tracking and disturbance rejection simultaneously where both the reference input and disturbances are generated by the autonomous system $\dot{v} = f_0(v)$ called exosystem.

- The problem can be viewed as a leader-following consensus problem with the exosystem as the leader and the controlled plant as the single follower.

- Two different approaches, namely, feedforward control and internal model control have been developed (Isidori, Huang, Khalil, et al).

- We have generalized both feedforward control and internal model control approaches to multi-agent systems since 2009. This talk will focus on feedforward control approach.
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    u = k(x, v)
\]

(1)

which leads to the following closed-loop system:

The solvability of the output regulation problem by feedforward control approach has been well understood after two decades of research (Isidori, Byrnes, Huang, et al).
Feedforward Control Approach

The feedforward control assumes $y = (x, v)$. The control law is of the following form:

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Observer based Control

The state $v(t)$ of the exosystem is often not available for control. It is desirable to design a control law which only depends on the measured output of the exosystem.

Given a system of the form

$$\dot{v} = f_0(v), \ y_o = g_o(v) \tag{2}$$

where $y_o$ is the measured output of (2). The following system

$$\dot{\eta} = \phi(\eta, y_o) \tag{3}$$

is called an asymptotic observer of (2) if, for any $v(0)$ and $\eta(0)$,

$$\lim_{t \to \infty} (\eta(t) - v(t)) = 0 \tag{4}$$

Observer based Control Law:

$$u = k(x, \eta), \ \dot{\eta} = \phi(\eta, y_o). \tag{5}$$
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is called an **asymptotic observer** of (2) if, for any \( v(0) \) and \( \eta(0) \),

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- Observer based Control Law:

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Certainty Equivalence Principle

- If an observer based control law (5) solves the same problem as the feedforward control law (1) does, then this control law is said to satisfy certainty equivalence principle.

- For linear time-invariant systems, an asymptotic observer exists generically, and the certainty equivalence principle always holds.

- For time-varying systems or nonlinear systems, an asymptotic observer may not exist, and even if it exists, the certainty equivalence principle may not hold. This is one of the reasons that makes the control of nonlinear or time-varying systems challenging.
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3. Cooperative Control of Multi-Agent Systems
Multi-agent Systems

\[
\begin{align*}
\dot{x}_i &= f_i(x_i, u_i, v, w) \\
e_i &= h_i(x_i, u_i, v, w), \quad i = 1, \ldots, N \\
y_i &= h_{mi}(x_i, u_i, v, w)
\end{align*}
\]

(6)

where \(x_i \in \mathbb{R}^{n_i}, u_i \in \mathbb{R}^{m_i}, e_i \in \mathbb{R}^p\), and \(y_i \in \mathbb{R}^{p_i}\). The exogenous signal \(v \in \mathbb{R}^q\) is generated by the following exosystem:

\[
\dot{v} = f_0(v), \quad y_0 = h_0(v)
\]

(7)

System (6) together with (7) can be viewed as a multi-agent system of \(N + 1\) agents where the exosystem (7) is viewed as the leader, and all subsystems of system (6) are viewed as \(N\) followers.

If all followers can access the state \(v\) of the leader, then the output regulation problem of (6) and (7) can be handled by a so-called decentralized control scheme.
Multi-agent Systems

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Decentralized Feedforward Control

Subsystem 1
\[ u_1 \rightarrow v(t) \rightarrow y_1 \]
\[ x_1 \rightarrow v(t) \rightarrow y_1 \]

Subsystem 2
\[ u_2 \rightarrow v(t) \rightarrow y_2 \]
\[ x_2 \rightarrow v(t) \rightarrow y_2 \]

Subsystem \( N \)
\[ u_N \rightarrow v(t) \rightarrow y_N \]
\[ x_N \rightarrow v(t) \rightarrow y_N \]

Leader System
\[ v(0) \]

Feedforward Controller 1

Feedforward Controller 2

Feedforward Controller \( N \)
Communication Graph

Given the multi-agent system (6) and (7), one can define a communication graph $\mathcal{G}(t) = \{\mathcal{V}, \mathcal{E}(t)\}$ with $\mathcal{V}$ being the node set and $\mathcal{E}(t)$ being the edge set.

- $\mathcal{V} = \{0, 1, \ldots, N\}$ with the node 0 associated with (7) and the other $N$ nodes associated with the $N$ followers of (6).

- For any $t \geq 0$, $\mathcal{E}(t) \subset \mathcal{V} \times \mathcal{V}$. $(j, i) \in \mathcal{E}(t)$, $i \neq j$, $i, j = 0, 1, \ldots, N$, if and only if the control $u_i$ of the subsystem $i$, $i = 1, \ldots, N$, can access $y_j$ at time $t$, $j = 0, 1, \ldots, N$. $j$ is said to be a neighbor of $i$ at time $t$.

- $\mathcal{N}_i(t) = \{j, (j, i) \in \mathcal{E}(t)\}$ denotes the neighbor set of the node $i$ at time $t$.

- $\mathcal{G}(t)$ is said to be static if $\mathcal{G}(t) = \mathcal{G}(0)$ for any $t \geq 0$. 
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A subset of $\mathcal{E}(t)$ of the form $\{(i_1, i_2), (i_2, i_3), \ldots, (i_{k-1}, i_k)\}$ is called a path of $\mathcal{G}(t)$ from $i_1$ to $i_k$ at time $t$, and it is said that the node $i_1$ can reach the node $i_k$ at time $t$.

If, at some $t$, the node 0 can reach all other nodes, then the graph $\mathcal{G}(t)$ is said to be connected at time $t$. 

![Graph Diagram]
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Distributed Control Laws

Distributed control law:

\[ u_i = k_i(z_i, y_i, y_j, j \in \mathcal{N}_i(t)) \]
\[ \dot{z}_i = g_i(z_i, y_i, y_j, j \in \mathcal{N}_i(t)), \quad i = 1, \ldots, N \]  \hspace{1cm} (8)

where \( y_0 = h_0(v) \), \( k_i \) and \( g_i \) are some sufficiently smooth functions.

Control law (8) satisfies the communication constraints: the \( i^{th} \) control \( u_i \) depends on \( y_j \) iff the agent \( j \) is a neighbor of the agent \( i \).
**Distributed Control Laws**

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\end{align*}
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where \( y_0 = h_0(v) \), \( k_i \) and \( g_i \) are some sufficiently smooth functions.

Control law (8) satisfies the communication constraints: the \( i^{th} \) control \( u_i \) depends on \( y_j \) iff the agent \( j \) is a **neighbor** of the agent \( i \).
Distributed Control Laws

Distributed control law:

\[ u_i = k_i(z_i, y_i, y_j, j \in \mathcal{N}_i(t)) \]
\[ \dot{z}_i = g_i(z_i, y_i, y_j, j \in \mathcal{N}_i(t)), \quad i = 1, \ldots, N \]

where \( y_0 = h_0(v) \), \( k_i \) and \( g_i \) are some sufficiently smooth functions.

Control law (8) satisfies the communication constraints: the \( i^{th} \) control \( u_i \) depends on \( y_j \) iff the agent \( j \) is a neighbor of the agent \( i \).
Problem Formulation

Definition: Given the plant, the exosystem, and the graph $G(t)$, find a distributed control law such that, for any initial condition, the solution of the closed-loop system is bounded, and the error output satisfies

$$
\lim_{t \to \infty} e_i(t) = 0, \quad i = 1, \ldots, N.
$$

Remark 1: The degree of the difficulty of the problem not only depends on the dynamics of the system, but also the property of the graph $G(t)$ which can be static or time-varying satisfying such conditions as every time connected, frequently connected, or jointly connected.

Remark 2: The jointly connected condition is the most challenging one since such a graph is discontinuous and can be disconnected at any time instant.
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4. A Distributed Observer Approach
A Distributed Observer Based Scheme

A Distributed Observer Approach

Observer based Controller 1

Observer based Controller 2

Observer based Controller N

Subsystem 1

Subsystem 2

Subsystem N

Distributed Observer

Leader System

\( \eta_1 \rightarrow v \text{ as } t \rightarrow \infty \)

\( \eta_2 \rightarrow v \text{ as } t \rightarrow \infty \)

\( \eta_N \rightarrow v \text{ as } t \rightarrow \infty \)

\( v(0) \)

\( v(t) \)

\( u_1 \)

\( u_2 \)

\( u_N \)

\( x_1 \)

\( x_2 \)

\( x_N \)

\( y_1 \)

\( y_2 \)

\( y_N \)
Two Technical Issues

➤ Does such a distributed observer exist?

➤ Does the certainty equivalence principle hold?
Two Technical Issues

- Does such a distributed observer exist?

- Does the certainty equivalence principle hold?
Distributed Observer Candidate

Given the leader system $\dot{v} = f_0(v), \ y_0 = h_0(v)$ and a graph $G(t)$ with $N+1$ nodes, for $i = 1, \ldots, N$, $j = 0, 1, \ldots, N$, let $a_{ij}(t) > 0$ if $j \in N_i(t)$, and $a_{ij}(t) = 0$ if otherwise. Then the following compensator

$$\dot{\eta}_i = f_0(\eta_i) + \mu \left( \sum_{j \in N_i(t)} a_{ij}(t)(\eta_j - \eta_i) \right), \ i = 1, \ldots, N \quad (9)$$

where $\mu > 0$, $\eta_0 = y_0$, is called a distributed observer candidate of the leader system, and is called a distributed observer of the leader if

$$\lim_{t \to \infty} (\eta_i(t) - v(t)) = 0, \ i = 1, \ldots, N \quad (10)$$

Whether or not (9) is a distributed observer depends on both the dynamics of the leader and the property of the graph.
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Distributed Observer Based Control Law

 ➢ Decentralized Control Law:

\[
\begin{align*}
    u_i &= k_i(z_i, y_i, v) \\
    \dot{z}_i &= g_i(z_i, y_i, v), \quad i = 1, \ldots, N
\end{align*}
\] (11)

 ➢ Distributed Observer Based Control Law:

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\begin{align*}
    u_i &= k_i(z_i, y_i, \eta_i), \quad \dot{z}_i = g_i(z_i, y_i, \eta_i) \\
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\end{align*}
\] (12)

➢ Whether or not (12) satisfies the certainty equivalence principle depends on both the dynamics of the leader and follower, and the property of the graph.
Distributed Observer Based Control Law

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Whether or not (12) satisfies the certainty equivalence principle depends on both the dynamics of the leader and follower, and the property of the graph.
Results on Linear Multi-agent Systems

Proposition 1: Suppose the leader system is linear, and marginally stable, and the graph $G(t)$ is jointly connected. Then there exists positive $\mu$ such that, for all $v(0), \eta_i(0), i = 1, \ldots, N$, the solution of (9) satisfies

$$\lim_{t \to \infty} (\eta_i(t) - v(t)) = 0$$

Thus, (9) is a distributed observer of the leader.

Theorem 1: Suppose the leader system is linear and marginally stable, the follower system is linear, and the graph $G(t)$ is jointly connected. Then the certainty equivalence principle holds.

Remark 3: Theorem 1 leads to the solution to the cooperative output regulation problem for heterogeneous linear multi-agent systems subject to parameter uncertainties, and external disturbances.
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Results on Multiple Euler-Lagrange Systems

Multiple Euler-Lagrange Systems (Li and Slotine, 1991):

\[ M_i(q_i)\ddot{q}_i + C_i(q_i, \dot{q}_i)\dot{q}_i + G_i(q_i) = \tau_i, \ i = 1, \ldots, N \]  \hspace{1cm} (13)

where \( q_i, \dot{q}_i \in R^n \) are the generalized position and velocity vectors, and \( \tau_i \in R^n \) is the control input.

Leader System:

\[ \dot{v} = Sv, \ q_0 = Fv \]  \hspace{1cm} (14)

where \( v \in R^m, q_0 \in R^n, S \in R^{m\times m} \) and \( F \in R^{n\times m} \).

Remark 4: System (14) can generate a large class of leader signals such as step function of arbitrary magnitude, ramp function of arbitrary slope, and sinusoidal function of arbitrary amplitude and initial phase.
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Results on Multiple Euler-Lagrange Systems (Cont)

Theorem 2: Suppose the system matrix $S$ of the leader system is marginally stable, and the graph $\mathcal{G}(t)$ is jointly connected. Then the certainty equivalence principle holds for multiple EL systems.

Remark 5: Existing results only allow the leader signals with the known boundary of the initial condition or rely on discontinuous control.

Remark 6: The establishment of this result needs to overcome two technical difficulties: the nonlinearity of EL systems and the discontinuity of the closed-loop system since the graph $\mathcal{G}(t)$ is discontinuous.
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Results on Multiple Rigid Body Systems

Multiple Rigid Body Systems (Sidi 1997):

\[
\begin{align*}
\dot{\hat{q}}_i &= \frac{1}{2} \hat{q}_i \times \omega_i + \frac{1}{2} \bar{q}_i \omega_i, \quad \dot{\bar{q}}_i = -\frac{1}{2} \hat{q}_i^T \omega_i \\
J_i \dot{\omega}_i &= -\omega_i \times J_i \omega_i + u_i, \quad i = 1, \ldots, N
\end{align*}
\] (15a)

where \( \hat{q}_i \in \mathbb{R}^3 \) and \( \bar{q}_i \in \mathbb{R} \), and \( q_i = [\hat{q}_i, \bar{q}_i]^T \) is the unit quaternion representing the attitude of the \( i^{th} \) rigid body, \( \omega_i \in \mathbb{R}^3 \) is the angular velocity of the \( i^{th} \) rigid body, and \( J_i \in \mathbb{R}^{3 \times 3} \) and \( u_i \in \mathbb{R}^3 \) denote the inertia matrix and the control torque of the \( i^{th} \) rigid body, respectively.

Leader System:

\[
\begin{align*}
\dot{\hat{q}}_0 &= \frac{1}{2} \hat{q}_0 \times \omega_0 + \frac{1}{2} \bar{q}_0 \omega_0, \quad \dot{\bar{q}}_0 = -\frac{1}{2} \hat{q}_0^T \omega_0 \\
\dot{\omega}_0 &= S \omega_0
\end{align*}
\]

where \( S \) is some marginally stable matrix.
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\dot{\bar{q}}_i &= -\frac{1}{2} \hat{q}_i^T \omega_i \tag{15a}
\end{align*}
\]

\[
J_i \dot{\omega}_i = -\omega_i \times J_i \omega_i + u_i, \quad i = 1, \ldots, N \tag{15b}
\]

where \( \hat{q}_i \in \mathbb{R}^3 \) and \( \bar{q}_i \in \mathbb{R} \), and \( q_i = [\hat{q}_i, \bar{q}_i]^T \) is the unit quaternion representing the attitude of the \( i^{th} \) rigid body, \( \omega_i \in \mathbb{R}^3 \) is the angular velocity of the \( i^{th} \) rigid body, and \( J_i \in \mathbb{R}^{3 \times 3} \) and \( u_i \in \mathbb{R}^3 \) denote the inertia matrix and the control torque of the \( i^{th} \) rigid body, respectively.

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\end{align*}
\]

where \( S \) is some marginally stable matrix.
Results on Multiple Rigid Body Systems (Cont)

Let \( v = (q_0, \omega_0) \), and

\[
f_0(v) = \begin{bmatrix}
\frac{1}{2} \hat{q}_0 \times \omega_0 + \frac{1}{2} \bar{q}_0 \omega_0 \\
\frac{1}{2} \hat{q}_0^T \omega_0 \\
-S\omega_0
\end{bmatrix}
\]

Then the leader system takes the following standard form

\[
\dot{v} = f_0(v), \quad y_0 = v
\]

(17)

**Proposition 2:** Given a nonlinear leader system of the form (17) and a static graph \( G \), there exists positive \( \mu \) such that, for all \( v(0), \eta_i(0), i = 1, \cdots, N \),

\[
\lim_{t \to \infty} (\eta_i(t) - v(t)) = 0
\]

iff the graph \( G \) is connected.
Results on Multiple Rigid Body Systems (Cont)

Let $v = (q_0, \omega_0)$, and

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Then the leader system takes the following standard form

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Results on Multiple Rigid Body Systems (Cont.)

- **Theorem 3:** Given a static graph $G$, the certainty equivalence principle holds for the multiple rigid body system iff the graph $G$ is connected.

- **Remark 7:** Existing results are local, only allow the desired angular velocity to be step function, and cannot handle model uncertainty.

- **Remark 8:** This theorem means that the separation principle works for the multiple spacecraft system even if the leader system and the follower system are nonlinear.
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➢ **Theorem 3**: Given a static graph $G$, the certainty equivalence principle holds for the multiple rigid body system iff the graph $G$ is connected.

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5. A Case Study: Attitude Consensus of Multiple Spacecraft Systems
Certainty Equivalence Controller

- True Tracking Error:
  \[ \epsilon_i = q_0^{-1} \odot q_i \]
  \[ \hat{\omega}_i = \omega_i - C_i \omega_0 \]

The error signals \( \epsilon_i \) and \( \hat{\omega}_i \) are not available for every follower.

- Estimated Tracking Error: Partition \( \eta_i = [\zeta_i^T, \xi_i^T] \) with \( \alpha_i \in \mathbb{R}^4 \). Then
  \[ e_i = \zeta_i^{-1} \odot q_i \quad \text{(18a)} \]
  \[ \bar{\omega}_i = \omega_i - \hat{C}_i \xi_i \quad \text{(18b)} \]

where \( \hat{C}_i = (\bar{e}_i^2 - \hat{e}_i^T \hat{e}_i)I_3 + 2\hat{e}_i \hat{e}_i^T - 2\bar{e}_i \hat{e}_i^X \)

- Theorem 3 implies that, \( \forall \ i = 1, \ldots, N \),
  \[ \lim_{t \to \infty} (e_i(t) - \epsilon_i(t)) = 0, \quad \lim_{t \to \infty} (\bar{\omega}_i(t) - \hat{\omega}_i(t)) = 0 \]
Certainty Equivalence Controller

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\[ e_i = \zeta_i^{-1} \odot q_i \]  \hspace{1cm} (18a)
\[ \bar{\omega}_i = \omega_i - \hat{C}_i \xi_i \] \hspace{1cm} (18b)

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- True Tracking Error:
  
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  The error signals \( \epsilon_i \) and \( \hat{\omega}_i \) are not available for every follower.

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  Then
  
  \[ e_i = \zeta_i^{-1} \circ q_i \]  \hspace{1cm} (18a)
  \[ \bar{\omega}_i = \omega_i - \hat{C}_i \xi_i \]  \hspace{1cm} (18b)

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- Theorem 3 implies that, \( \forall \ i = 1, \ldots, N \),
  
  \[ \lim_{t \to \infty} (e_i(t) - \epsilon_i(t)) = 0, \quad \lim_{t \to \infty} (\bar{\omega}_i(t) - \hat{\omega}_i(t)) = 0 \]
Certainty Equivalence Controller

Decentralized Controller:

\[ u_i = \omega_i \times J_i \omega_i - J_i (\tilde{\omega}_i \times C_i \omega_i - C_i S \omega_i) - k_1 i \hat{e}_i - k_2 i \hat{\omega}_i, \quad i = 1, 2, \ldots, N \]

where \( k_{1i}, k_{2i} > 0 \).

Certainty Equivalence Controller:

\[ u_i = \omega_i \times J_i \omega_i - J_i (\tilde{\omega}_i \times \hat{C}_i \xi_i - \hat{C}_i S \xi_i) - k_1 i \hat{e}_i - k_2 i \tilde{\omega}_i \quad (19) \]

\[ \dot{\eta}_i = f_0(\eta_i) + \mu \left( \sum_{j \in N_i(t)} a_{ij}(t)(\eta_j - \eta_i) \right), \quad i = 1, \ldots, N \quad (20) \]

\( J_i \) is uncertain due to
- uncertain mass distribution;
- fuel consumption;
- spacecraft reconfiguration;

Therefore, the control law \( u_i \) must not rely on \( J_i \). Adaptive control is such a control scheme.
The Error System

To simplify the closed-loop system analysis, performing on (18a) the following transformation

\[ \tilde{\omega}_i = \bar{\omega}_i + k_{i1}\hat{e}_i \]

where \( k_{i1} > 0 \), leads to the following error system:

\[ \begin{align*}
\dot{\hat{e}}_i &= \frac{1}{2}(\hat{e}_i^\times + \bar{e}_iI_3)(\tilde{\omega}_i - k_{i1}\hat{e}_i) + \alpha_i(t) \\
\dot{\bar{e}}_i &= -\frac{1}{2}\hat{e}_i^T(\tilde{\omega}_i - k_{i1}\hat{e}_i) + \beta_i(t) \\
J_i\dot{\bar{\omega}}_i &= -\omega_i^\times J_i\omega_i + J_i((\tilde{\omega}_i - k_{i1}\hat{e}_i)^\times \hat{C}_i\xi_i - \hat{C}_iS\xi_i) \\
&\quad + \frac{1}{2}k_{i1}(\hat{e}_i^\times + \bar{e}_iI_3)(\tilde{\omega}_i - k_{i1}\hat{e}_i)) + \gamma_i(t) + u_i
\end{align*} \]  

(21a)

(21b)

(21c)

where \( \alpha_i(t) \), \( \beta_i(t) \) and \( \gamma_i(t) \) satisfy

\[ \lim_{t \to \infty} \alpha_i(t) = 0, \quad \lim_{t \to \infty} \beta_i(t) = 0, \quad \lim_{t \to \infty} \gamma_i(t) = 0 \]
Simplification

Objective of Control: \( \forall i = 1, \ldots, N, \lim_{t \to \infty} \hat{e}_i(t) = 0 \) and 
\( \lim_{t \to \infty} \tilde{\omega}_i(t) = 0. \)

Lemma 1: Consider (21a) and (21b). If \( \tilde{\omega}_i(t) \) is piecewise continuous for \( t \geq 0 \), and
\[
\lim_{t \to \infty} \tilde{\omega}_i(t) = 0, \ i = 1, \ldots, N,
\]
then \( e_i(t) \) is bounded for all \( t \geq 0 \) and
\[
\lim_{t \to \infty} \hat{e}_i(t) = 0, \ i = 1, \ldots, N.
\]

Lemma 2: The leader-following consensus problem is solvable by a distributed control law if the same control law renders
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Simplification

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By Lemma 2, it suffices to design a distributed control law of the form (12) to globally stabilize the following error dynamic equation

\[ J_i \dot{\hat{\omega}}_i = -\omega_i \times J_i \omega_i + J_i ((\tilde{\omega}_i - k_{i1} \hat{e}_i) \times \hat{C}_i \xi_i - \hat{C}_i S \xi_i + \frac{1}{2} k_{i1} (\hat{e}_i \times + \bar{e}_i I_3) (\tilde{\omega}_i - k_{i1} \hat{e}_i)) + \gamma_i(t) + u_i \]  

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To apply the adaptive control technique to system (22), we need to put equation (22) in the standard form where the unknown parameters appear linearly.
By Lemma 2, it suffices to design a distributed control law of the form (12) to globally stabilize the following error dynamic equation

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**Linear Parameterization**

For any \( x = [x_1 \ x_2 \ x_3]^T \in \mathbb{R}^3 \), define a linear operator \( L \) acting on \( x \) by

\[
L(x) = \begin{bmatrix}
    x_1 & 0 & 0 & 0 & x_3 & x_2 \\
    0 & x_2 & 0 & x_3 & 0 & x_1 \\
    0 & 0 & x_3 & x_2 & x_1 & 0
\end{bmatrix}.
\]

Let \( J_i \) be denoted by

\[
J_i = \begin{bmatrix}
    J_{i11} & J_{i12} & J_{i13} \\
    J_{i12} & J_{i22} & J_{i23} \\
    J_{i13} & J_{i23} & J_{i33}
\end{bmatrix},
\]

and define

\[
\Theta_i = [J_{i11} \ J_{i22} \ J_{i33} \ J_{i23} \ J_{i13} \ J_{i12}]^T.
\]

Then

\[
J_i x = L(x) \Theta_i.
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Thus equation (22) can be rewritten as

\[ J_i \dot{\omega}_i = \psi_i(t) \Theta_i + \gamma_i(t) + u_i \]  

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where

\[ \psi_i(t) = -\omega_i \times L(\omega_i) + L((\omega_i - k_i \hat{e}_i) \times \hat{\hat{C}}_i \xi_i - \hat{C}_i S \xi_i \]

\[ + \frac{1}{2} k_i (\hat{e}_i^2 + \hat{e}_i I_3) (\omega_i - k_i \hat{e}_i) ] \]

\[ \psi_i(t) \] is independent of \((q_0, \omega_0)\) whenever \(0 \notin N_i\).
Thus equation (22) can be rewritten as

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$\psi_i(t)$ is independent of $(q_0, \omega_0)$ whenever $0 \notin \mathcal{N}_i$. 
Distributed Adaptive Control Law

- If $\gamma_i(t)$ is identically zero for all $t \geq 0$, then (23) is in the same form as what was studied in [Chen and Huang 2009] where it was shown that (23) can be globally stabilized by the following adaptive control law:

$$
\begin{align*}
    u_i &= -\psi_i(t)\hat{\Theta}_i - k_{i2}\tilde{\omega}_i, \\
    \dot{\hat{\Theta}}_i &= \Lambda_i^{-1}\psi_i(t)^T\tilde{\omega}_i, \\
    i &= 1, \cdots, N
\end{align*}
$$

(24)

where $k_{i2} > 0$, $\Lambda_i \in \mathbb{R}^{6 \times 6}$ is some positive definite gain matrix.

- It turns out that, when $\gamma_i(t)$ is not identically zero, but $\lim_{t \to \infty} \gamma_i(t) = 0$, the same control law (24) also globally stabilize (23).

- This control law together with the distributed observer (9) constitutes a distributed adaptive control law of the form (12).
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An Example

Communication network with one leader and four followers:
An Example (Cont.)

Desirable Angular Velocity

$\Omega_0$: Let

$$\omega_0(t) = [\sin t, \cos t, 3]^T$$

which can be produced by the leader system with

$$S = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and $\omega_0(0) = [0, 1, 3]^T$

Initial orientation of the leader:

$$q_0(0) = [0 \ 0 \ 0 \ 1]^T$$
An Example (Cont.)

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6. Concluding Remarks

➢ This talk has presented a framework for handling the cooperative control problem of multi-agent systems via the distributed observer approach.

➢ Under the assumption that the graph is connected, a distributed observer based controller = a decentralized controller + a distributed observer.

➢ A distributed observer based controller satisfies certainty equivalence principle if it does what a decentralized controller does.
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- Under the assumption that the graph is connected, a distributed observer based controller = a decentralized controller + a distributed observer.

- A distributed observer based controller satisfies certainty equivalence principle if it does what a decentralized controller does.
The framework has led to a complete solution to the cooperative output regulation problem for general, heterogeneous, uncertain linear multi-agent systems subject to external disturbances.

It also applies to two classes of practical nonlinear systems, namely, Euler-Lagrange systems and rigid body systems.

The approach has also been applied to flocking, formation, rendezvous, etc., of some classes of linear systems and Euler-Lagrange systems, thus making the graph connectivity an objective instead of an assumption.

The cooperative control problem for more general uncertain nonlinear systems can be handled by internal model based approach.
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Thanks

Thank you!