OUTPUT CONSENSUS OF HETEROGENEOUS LINEAR MULTI-AGENT SYSTEMS BY EVENT-TRIGGERED CONTROL

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Introduction
- Background
- Problem Statement

Research Methodologies
- Main Challenges
- Internal Reference Model
- Event-Triggered Control for Homogeneous Systems

Main Results
- A Sufficient and Necessary Condition
- Event-Triggered Control Design
- Feasibility
- Self-Triggered Control Design

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- System Model
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Introduction

Background

Multi-Agent System (MAS)

- Modeling/describing some **collective behaviors** of some animals.
- Wide applications in **engineering problems**.

Some engineering applications of the MAS.

(\url{http://www.ri.cmu.edu/research_guide/multi_agent_systems.html})

- One fundamental problem: **consensus problem**.
Two Kinds of MASs

- Early researches focused on MAS with identical dynamics.

\[ \dot{x}_i = Ax_i + Bu_i, \quad i = 1, \ldots, N, \quad (1.1) \]

—called **homogeneous** MAS.

- In many applications, the agents’ dynamics are non-identical.

\[ \begin{align*}
\dot{x}_i &= A_ix_i + B_iu_i \\
y_i &= C_ix_i, \quad i = 1, \ldots, N,
\end{align*} \quad (1.2) \]

—called **heterogeneous** MAS.

- All agents communicate with each other through a communication graph \( G \).
Consensus Problem

- **Homogeneous MAS:** state consensus problem.
  Definition: Design $u_i$ such that
  \[
  \lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0, \quad \forall i, j = 1, \cdots, N, \tag{1.3}
  \]
  holds for any finite $x_i(0), \forall i = 1, \cdots, N$.

- **Heterogeneous MAS:** output consensus problem.
  Definition: Design $u_i$ such that
  \[
  \lim_{t \to \infty} \|y_i(t) - y_j(t)\| = 0, \quad \forall i, j = 1, \cdots, N, \tag{1.4}
  \]
  holds for any finite $x_i(0), \forall i = 1, \cdots, N$.

- **Remark 1:** Output consensus problem includes state consensus problem as a special case.
Why Event-Triggered Strategy?

- Individual agents equipped with microprocessors and some actuation modules.
  - On-board energy and resources are limited.
  - Energy-saving control schemes are needed.

- To reduce the communication load.
  - Proposed in stabilization problem for a single system [Tabuada (2007)].
  - Mainly applied to some MASs with simple agent dynamics.
Design an event-triggered control scheme, such that the output consensus problem of heterogeneous MAS (1.2) can be solved.

- Control input $u_i$ can only access information from itself and its neighboring agents.
- Event-triggered strategy should be integrated in the control scheme.
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Main Challenges

- **Heterogeneity Problem.**
  - Different dynamics: \( A_i \neq A_j, \ B_i \neq B_j \) for \( i \neq j \).
  - Different state dimensions.

- **Use of event-triggered strategy.**
  - Convergence analysis.
  - Development of event-triggering condition based on local information.
  - Feasibility analysis.
Introduction of internal reference models.

\[ \dot{\eta}_i = S\eta_i + \tilde{u}_i \]

\[ \dot{x}_i = A_ix_i + B_iu_i \]

The internal reference models of the heterogeneous MAS.
Key Idea

- Designed to be of identical dynamics.

\[
\dot{\eta}_i = S\eta_i + \tilde{u}_i. \tag{2.1}
\]

- Regarded as a homogeneous MAS.
  - Exchanging information: states of the internal reference models.
  - To reduce the communication load: event-triggered strategy.

- Objective: \((\eta_i - \eta_0) \to 0 \text{ as } t \to \infty, \forall i = 1, \ldots, N.\)

- \(\eta_0\) can be regarded as a reference signal for each agent.
Event-Triggered Control for Homogeneous MAS

- For a homogeneous MAS

\[ \dot{x}_i = Ax_i + B\tilde{u}_i, \quad i = 1, \ldots, N, \quad (2.2) \]

- Define the combined measurement as

\[ \tilde{q}_i(t) = \sum_{j=1}^{N} a_{ij} (x_j(t) - x_i(t)). \quad (2.3) \]

- Define the measurement error as

\[ \tilde{e}_i(t) = \tilde{q}_i(t^i_k) - \tilde{q}_i(t). \quad (2.4) \]

- Control law for each agent:

\[ \tilde{u}_i(t) = \tilde{K}\tilde{q}_i(t^i_k), \quad t \in [t^i_k, t^i_{k+1}), \quad (2.5) \]

- Triggering condition

\[ h(\tilde{e}_i(t), \tilde{q}_i(t)) = 0. \quad (2.6) \]
A Previous Result

**Lemma 1 [Hu et al. (2014)]:** Under the assumptions that \((A, B)\) is stabilizable and the undirected communication graph \(G\) is connected, there always exists at least one solution \(P > 0\) for the following inequality

\[
PA + A^T P - \alpha PBB^T P + \beta I_n \leq 0,
\]  

where \(0 < \alpha \leq 2\lambda_2, \beta \geq 2\lambda_N\), with \(\lambda_2\) and \(\lambda_N\) the Fiedler eigenvalue and the largest eigenvalue of the Laplacian matrix of \(G\), respectively. Then, letting \(\tilde{K} = B^T P\), the state consensus of homogeneous multi-agent system (2.2) can be achieved by the control law (2.5) and the following triggering condition

\[
h(\tilde{e}_i(t), \tilde{q}_i(t)) = \|\tilde{e}_i(t)\| - \tilde{\gamma}_i \|\tilde{q}_i(t)\| = 0.
\]  

where \(\tilde{\gamma}_i = \sqrt{\frac{\sigma_i \cdot a(2-a \rho)}{\rho}}\) with \(\sigma_i \in (0, 1), \rho = \|PBB^T P\|\), and \(a\) being a positive number satisfying \(a < \frac{2}{\rho}\).
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Main Results

A Sufficient and Necessary Condition

Event-Triggered Control Scheme

- Output feedback controller.

\[
\dot{\eta}_i = S\eta_i + K \sum_{j=1}^{N} a_{ij} (\eta_j(t^i_k) - \eta_i(t^i_k)) , \quad t \in [t^i_k, t^i_{k+1})
\]

\[
\dot{\xi}_i = A_i \xi_i + B_i u_i + H_i (C_i \xi_i - y_i)
\]

\[
u_i = K_{1i} \xi_i + K_{2i} \eta_i, \quad i = 1, \ldots, N.
\] (3.1)

Design parameters: \(S, K, H_i, K_{1i}, K_{2i}\); Triggering time sequence: \(\{t_0^i, t_1^i, \ldots\}\).

- Event-triggering condition.

\[
h(e_i(t), q_i(t)) = \|e_i(t)\| - \gamma_i \|q_i(t)\| = 0,
\] (3.2)

where \(q_i(t) = \sum_{j\in\mathcal{N}_i} (\eta_j(t) - \eta_i(t))\), \(e_i(t) = q_i(t^i_k) - q_i(t)\) and \(\gamma_i\) can be calculated by utilizing Lemma 1.

- Questions: Under what condition, the problem can be solved by (3.1) and (3.2)? How to design those parameters?
Assumptions & Result

- Assumptions:
  - Each pair of $(A_i, B_i)$ is stabilizable.
  - Each pair of $(A_i, C_i)$ is detectable.
  - The undirected communication graph $G$ is connected.

- **Theorem 1**: Consider the heterogeneous linear multi-agent system (1.2) under Assumptions 1-3. The output consensus problem can be solved by the proposed controller (3.1) with the triggering condition (3.2) if and only if there exists $(S, R)$, such that the following equations have solutions $(\Pi_i, \Gamma_i)$ for $i = 1, \cdots, N$, where $S, R, \Pi_i$ and $\Gamma_i$ all have compatible dimensions,

  \[ A_i\Pi_i + B_i\Gamma_i = \Pi_iS. \]  
  \[ C_i\Pi_i = R. \]
Remark 2: Internal reference model is used to generate a virtual reference signal $\eta_0$ for each agent.

The dynamics and output of the virtual reference signal

\[
\dot{\eta}_0(t) = S\eta_0(t) \\
y(t) = R\eta_0(t)
\]

with $(y_i(t) - y(t)) \to 0$ as $t \to \infty$, $\forall i = 1, \cdots, N$. 

Notes
Parameters Design

- **Design procedure.**
  - Step 1: Choose proper matrices $S$ and $R$, such that (3.3) and (3.4) have solution pairs $(\Pi_i, \Gamma_i), \ i = 1, \cdots, N$.
  - Step 2: Choose proper $\Lambda_i$, such that $A_i + B_i \Lambda_i$ is Hurwitz. Let $K_{1i}, K_{2i}$ be as follows,
    \[ K_{1i} = \Lambda_i, \ K_{2i} = \Gamma_i - \Lambda_i \Pi_i. \]  \tag{3.5}
  - Step 3: Choose a proper $H_i$, such that $A_i + H_i C_i$ is Hurwitz.
  - Step 4: Let $K = P$, where $P > 0$ satisfies the following inequality
    \[ PS + S^T P - \alpha PP + \beta I_m \leq 0. \]  \tag{3.6}
  - Step 5: Let $t_0^i, t_1^i, \cdots, \forall i = 1, \cdots, N$, to be determined by the proposed triggering condition (3.2).
Exclusion of Singular Triggering

- **Singular Triggering.**
  - Definition:
    - Method: To prove that if $t_i^k$ exists and $q_i(t_i^k) \neq 0$, the next triggering time $t_{i+1}^k$ exists and $q_i(t_{i+1}^k) \neq 0$.
  - Requirement: The proof holds for any agent.

- **Corollary 1**: Consider the heterogeneous multi-agent system (1.2) and the control scheme (3.1). No agent will exhibit singular triggering behavior.
Exclusion of Zeno behavior

- **Zeno behavior.**
  - **Definition:**
    
    ![Diagram](image)

    - Method: To prove that the length of inter-event interval is strictly positive.
    - Requirement: The proof holds for any agent.

- **Corollary 2:** Consider the heterogeneous multi-agent system (1.2) and the control scheme (3.1). **No agent will exhibit Zeno behavior.**
  - Calculate the inter-event interval for $i$th agent
    \[
    t_{k+1}^i - t_k^i > \frac{1}{\|S\|} \ln \left( \frac{\|S\| s_k^i}{\alpha_k^i} + 1 \right) \geq 0. \tag{3.7}
    \]
    where $s_k^i$ and $\alpha_k^i$ are two positive constants.
Main Results

Self-Triggered Control Design

Drawback of Event-Triggered Strategy

- In the proposed event-triggering condition
  \[ h(e_i(t), q_i(t)) = \|e_i(t)\| - \gamma_i \|q_i(t)\| = 0, \]
  continuous monitoring of \( e_i(t) \) and \( q_i(t) \) are required.

- How to avoid this constraint?
Self-Triggered Strategy

- Main idea: Estimate the next triggering time based on the measurement at previous triggering time.

- We propose the following condition

\[ \| e_i(t) \| \leq \frac{\gamma_i}{\sqrt{2 + 2\gamma_i^2}} \| q_i(t_k^i) \| = s_k^i \]

which implies \( h(e_i(t), q_i(t)) \leq 0 \).

- Calculate the time it will be needed for \( \| e_i(t) \| \) to increase to \( s_k^i \).
Self-Triggering Condition

- Calculate the increasing rate of $\|e_i(t)\|$. 

$$\frac{d}{dt} \|e_i(t)\| \leq \|S\| s_k^i + w_i(t),$$

where $w_i(t) = \left\| (d_i P - S) q_i(t_k^i) - P \sum_{j \in \mathcal{N}_i} q_j(t_{k'}^i(t))] \right\|$, with $t_{k'}^i(t)$ being the latest triggering time for agent $j$, $j \in \mathcal{N}_i$.

- **Self-Triggering Rule**
  
  If no neighboring agent is triggered ahead of agent $i$, then $t_{k+1}^i = \frac{s_k^i}{\|S\| s_k^i + w_i(t_k^i)}$. 
  
  Otherwise, if one neighboring agent $j$ is triggered first at time $t'$, then update $w_i(t)$ as $w_i(t')$ and calculate the left time which will be needed for $\|e_i(t)\|$ to increase to $s_k^i$.

- The feasibility of self-triggering rule: omitted here.
Theorem 2: Consider the heterogeneous linear multi-agent system (1.2) under Assumptions 1-3. The output consensus problem can be solved by the proposed controller (3.1) and the self-triggering rule if and only if there exists \((S, R)\), such that (3.3) and (3.4) always have solutions \((\Pi_i, \Gamma_i)\) for \(i = 1, \cdots, N\), where \(S, R, \Pi_i\) and \(\Gamma_i\) all have compatible dimensions.
An Example

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Consider the following heterogeneous MAS [Wieland et al. (2011)].

\[
\dot{x}_i = \begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & c_i \\
0 & -d_i & -a_i \\
\end{pmatrix} x_i + \begin{pmatrix}
0 \\
0 \\
b_i \\
\end{pmatrix} u_i
\]

\[
y_i = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
\end{pmatrix} x_i, \quad i = 1, 2, 3, 4,
\]

where parameters \( \{a_i, b_i, c_i, d_i\} \) are set as \( \{1, 1, 1, 0\} \), \( \{10, 2, 1, 0\} \), \( \{2, 1, 1, 10\} \) and \( \{2, 1, 1, 1\} \), respectively.

The communication graph.

The communication graph \( G \).
Event-triggered control scheme is designed according to Steps 1-5.

- In Step 1:
  \[ S = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, R = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \]

- In Step 2:
  \[ \Lambda_i = \begin{pmatrix} -1 & -1 & -1 \end{pmatrix}, \quad K_{1i} = \Lambda_i = \begin{pmatrix} -1 & -1 & -1 \end{pmatrix}, \]
  \[ K_{2i} = \Gamma_i - \Lambda_i \Pi_i = \begin{pmatrix} 1 & 1 + \frac{d_i}{b_i} \end{pmatrix}. \]

- In Step 3: We set \( H_i = \begin{pmatrix} 0 & 0 \\ -10 & -10 \\ 9 & 9 \end{pmatrix}, i = 1, \cdots, 4. \)

- In Step 4: \( K = P = \begin{pmatrix} 1.9848 & 0.2462 \\ 0.2462 & 2.0459 \end{pmatrix}. \)

- In Step 5: Threshold in the triggering condition can be calculated \( \gamma_i = 0.1942. \) Triggering time sequence can be calculated by event-triggering condition or self-triggering rule.
Simulation results.

Output response of all agents via event-triggered control scheme.

Note: \( y_i = \text{col}(y_{i1}, y_{i2}) \) and \( e^y_{ij} = y_{i1} - y_{j1} \).
Simulation results.

Output response of all agents via self-triggered control scheme.

Note: \( y_i = \text{col}(y_{i1}, y_{i2}) \) and \( e^y_{ij} = y_{i1} - y_{j1} \).
Performances comparison between two proposed control schemes

<table>
<thead>
<tr>
<th>Control scheme</th>
<th>$T_s$ (sec)</th>
<th>Triggering numbers for agents</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>Event-triggered</td>
<td>191.97</td>
<td>249</td>
</tr>
<tr>
<td>Self-triggered</td>
<td>85.08</td>
<td>805</td>
</tr>
</tbody>
</table>

where $T_s$ is defined as a minimum time, such that, $\|y_i(t) - y_j(t)\| \leq 0.001$ when $t \geq T_s$, for any agents $i,j$.

Less settling time while more triggering numbers are needed to reach output consensus by the self-triggered control scheme.
Conclusions

- Output consensus problem of heterogeneous linear MASs has been studied.
- A novel event-triggered control scheme has been proposed.
- Feasibility of the proposed control scheme has been discussed.
- A novel self-triggered control scheme has been proposed.


Thank You!